Hydrology: Frequency Analysis of Flood Data

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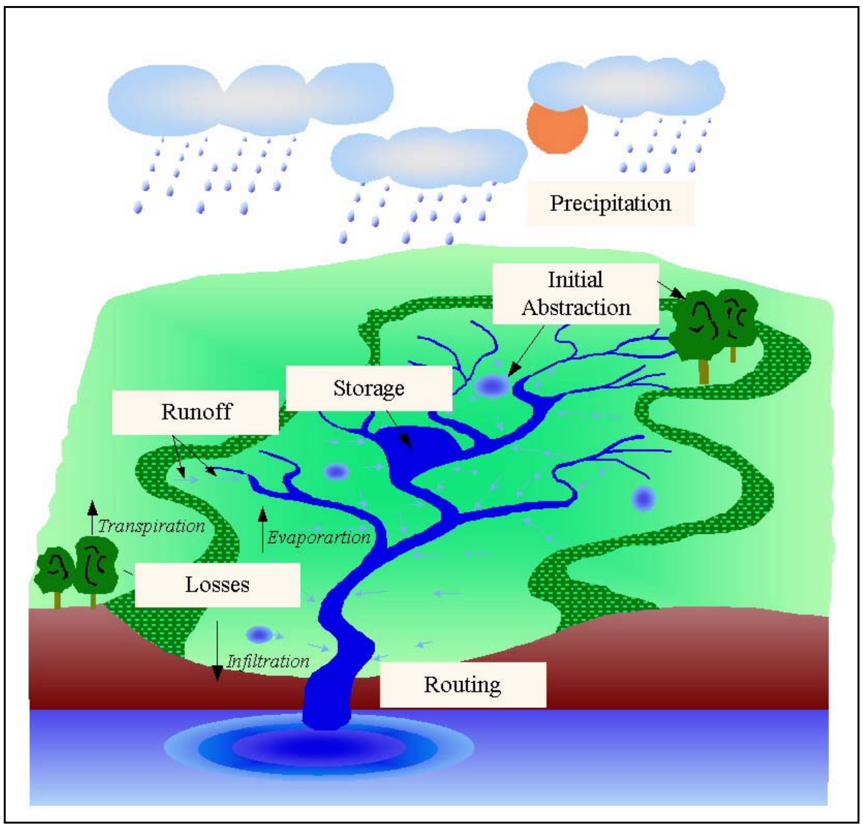

National Highway Institute

CHAPTER 4

PEAK FLOW FOR GAGED SITES

The estimation of peak discharges of various recurrence intervals is one of the most common problems faced by engineers when designing for highway drainage structures. The problem can be divided into two categories:

- Gaged sites: the site is at or near a gaging station, and the stream flow record is fairly complete and of sufficient length to be used to provide estimates of peak discharges.

- Ungaged sites: the site is not near a gaging station or the stream flow record is not adequate for analysis.

Sites that are located at or near a gaging station, but that have incomplete or very short records represent special cases. For these situations, peak discharges for selected frequencies are estimated either by supplementing or transposing data and treating them as gaged sites; or by using regression equations or other synthetic methods applicable to ungaged sites.

The USGS Interagency Advisory Committee on Water Data Bulletin 17B (1982) is a guide that "describes the data and procedures for computing flood flow frequency curves where systematic stream gaging records of sufficient length (at least 10 years) to warrant statistical analysis are available as the basis for determination." The guide was intended for use in analyzing records of annual flood peak discharges, including both systematic records and historic data. The document iscommonly referred to simply as "Bulletin 17B".

Methods for making flood peak estimates can be separated on the basis of the gaged vs. ungaged classification. If gaged data are available at or near the site of interest, the statistical analysis of the gaged data is generally the preferred method of analysis. Where such data are not available, estimates of flood peaks can be made using either regional regression equations or one of the generally available empirical equations. If the assumptions that underlie the regional regression equations are valid for the site of interest, their use is preferred to the use of empirical equations. The USGS has developed and published regional regression equations for estimating the magnitude and frequency of flood discharges for all states and the Commonwealth of Puerto Rico (Jennings, et al., 1994). Empirical approaches include the rational equation and the SCS graphical peak discharge equation.

This chapter is concerned primarily with the statistical analysis of gaged data. Appropriate solution techniques are presented and the assumptions and limitations of each are discussed. Regional regression equations and the empirical equations applicable to ungaged sites are discussed in Chapter 5.

4.1 RECORD LENGTH REQUIREMENTS

Analysis of gaged data permits an estimate of the peak discharge in terms of its probability or frequency of exceedence at a given site. This is done by statistical methods provided sufficient data are available at the site to permit a meaningful statistical analysis to be made. Bulletin 17B (1982) suggests that at least 10 years of record are necessary to warrant a statistical analysis by methods presented therein.

At some sites, historical data may exist on large floods prior to or after the period over which stream flow data were collected. This information can be collected from inquiries, newspaper accounts, and field surveys for highwater marks. Whenever possible, these data should be compiled and documented to improve frequency estimates.

4.2 STATISTICAL CHARACTER OF FLOODS

The concepts of populations and samples are fundamental to statistical analysis. A population that may be either finite or infinite is defined as the entire collection of all possible occurrences of a given quantity. An example of a finite population is the number of possible outcomes of the throw of the dice, a fixed number. An example of an infinite population is the number of different peak annual discharges possible for a given stream.

A sample is defined as part of a population. In all practical instances, hydrologic data are analyzed as a sample of an infinite population, and it is usually assumed that the sample is representative of its parent population. By representative, it is meant that the characteristics of the sample, such as its measures of central tendency and its frequency distribution, are the same as that of the parent population.

An entire branch of statistics deals with the inference of population characteristics and parameters from the characteristics of samples. The techniques of inferential statistics, which is the name of this branch of statistics, are very useful in the analysis of hydrologic data because samples are used to predict the characteristics of the populations. Not only will the techniques of inferential statistics allow estimates of the characteristics of the population from samples, but they also permit the evaluation of the reliability or accuracy of the estimates. Some of the methods available for the analysis of data are discussed below and illustrated with actual peak flow data.

Before analyzing data, it is necessary that they be arranged in a systematic manner. Data can be arranged in a number of ways, depending on the specific characteristics that are to be examined. An arrangement of data by a specific characteristic is called a distribution or a series. Some common types of data groupings are the following: magnitude; time of occurrence; and geographic location.

4.2.1 Analysis of Annual and Partial-Duration Series

The most common arrangement of hydrologic data is by magnitude of the annual peak discharge. This arrangement is called an annual series. As an example of an annual series, 29 annual peak discharges for Mono Creek near Vermilion Valley, California, are listed in Table 4.1.

Another method used in flood data arrangement is the partial-duration series. This procedure uses all peak flows above some base value. For example, the partial-duration series may consider all flows above the discharge of approximately bankfull stage. The USGS sets the base for the partial-duration series so that approximately three peak flows, on average, exceed the base each year. Over a 20-year period of record, this may yield 60 or more floods compared to 20 floods in the annual series. The record contains both annual peaks and partial-duration peaks for unregulated watersheds. Figure 4.1 illustrates a portion of the record for Mono Creek containing both the highest annual floods and other large secondary floods.

Table 4.1. Analysis of Annual Flood Series, Mono Creek, CA

Basin: Mono Creek near Vermilion Valley, CA, South Fork of San Joaquin River Basin

Location: Latitude 37°22'00", Longitude 118° 59' 20", 1.6 km (1 mi) downstream from lower end of Vermilion Valley and 9.6 km (6.0 mi) downstream from North Fork

Area: 238.3 km^2 (92 mi^2)

Remarks: diversion or regulation

Record: 1922-1950, 29 years (no data adjustments)

Year	Annual Maximum (m^3/s)	Smoothed Series (m^3/s)	Annual Maximum (ft^3/s)	Smoothed Series (ft^3/s)
1922	39.4	-	1,390	-
1923	26.6	-	940	-
1924	13.8	27.8	488	982
1925	30.0	28.0	1,060	988
1926	29.2	28.9	1,030	1,022
1927	40.2	30.4	1,420	1,074
1928	31.4	29.2	1,110	1,031
1929	21.2	26.4	750	931
1930	24.0	26.4	848	931
1931	14.9	27.7	525	979
1932	40.2	25.8	1,420	909
1933	38.2	27.9	1,350	986
1934	11.4	30.9	404	1,093
1935	34.8	29.8	1,230	1,051
1936	30.0	32.1	1,060	1,133
1937	34.3	32.8	1,210	1,160
1938	49.8	32.3	1,760	1,140
1939	15.3	34.3	540	1,212
1940	32.0	34.1	1,130	1,204
1941	40.2	32.3	1,420	1,140
1942	33.1	34.1	1,170	1,203
1943	40.8	35.4	1,440	1,251
1944	24.2	32.5	855	1,149
1945	38.8	31.5	1,370	1,113
1946	25.8	28.1	910	992
1947	28.0	28.4	988	1,004
1948	23.7	26.9	838	950
1949	25.9	-	916	-
1950	31.2	-	1,100	-

Partial-duration series are used primarily in defining annual flood damages when more than one event that causes flood damages can occur in any year. If the base for the partial-duration series conforms approximately to bankfull stage, the peaks above the base are generally flood-damaging events. The partial-duration series avoids a problem with the annual-maximum series, specifically that annual-maximum series analyses ignore floods that are not the highest flood of that year even though they are larger than the highest floods of other years. While partial-duration series produce larger sample sizes than annual maximum series, they require a criterion that defines peak independence. Two large peaks that are several days apart and separated by a period of lower flows may be part of the same hydrometeorological event and, thus, they may not be independent events. Independence of events is a basic assumption that underlies the method of analysis.

If these floods are ordered in the same manner as in an annual series, they can be plotted as illustrated in Figure 4.2. By separating out the peak annual flows, the two series can be compared as also shown in Figure 4.2, where it is seen that, for a given rank (from largest to smallest) order, m, the partial-duration series yields a higher peak flow than the annual series. The difference is greatest at the lower flows and becomes very small at the higher peak discharges. If the recurrence interval of these peak flows is computed as the rank order divided by the number of events (not years), the recurrence interval of the partial-duration series can be computed in the terms of the annual series by the equation:

$$T_B = \frac{1}{\ln T_A \;\; \ln(T_A - 1)} \tag{4.1}$$

where T_B and T_A are the recurrence intervals of the partial-duration series and annual series, respectively. Equation 4.1 can also be plotted as shown in Figure 4.3.

This curve shows that the maximum deviation between the two series occurs for flows with recurrence intervals less than 10 years. At this interval, the deviation is about 5 percent and, for the 5-year discharge, the deviation is about 10 percent. For the less frequent floods, the two series approach one another (see Table 4.2).

When using the partial-duration series, one must be especially careful that the selected flood peaks are independent events. This is a tough practical problem since secondary flood peaks may occur during the same flood as a result of high antecedent moisture conditions. In this case, the secondary flood is not an independent event. One should also be cautious with the choice of the lower limit or base flood since it directly affects the computation of the properties of the distribution (i.e., the mean, the variance and standard deviation, and the coefficient of skew), all of which may change the peak flow determinations. For this reason, it is probably best to utilize the annual series and convert the results to a partial-duration series through use of Equation 4.1. For the less frequent events (greater than 5 to 10 years), the annual series is entirely appropriate and no other analysis is required.

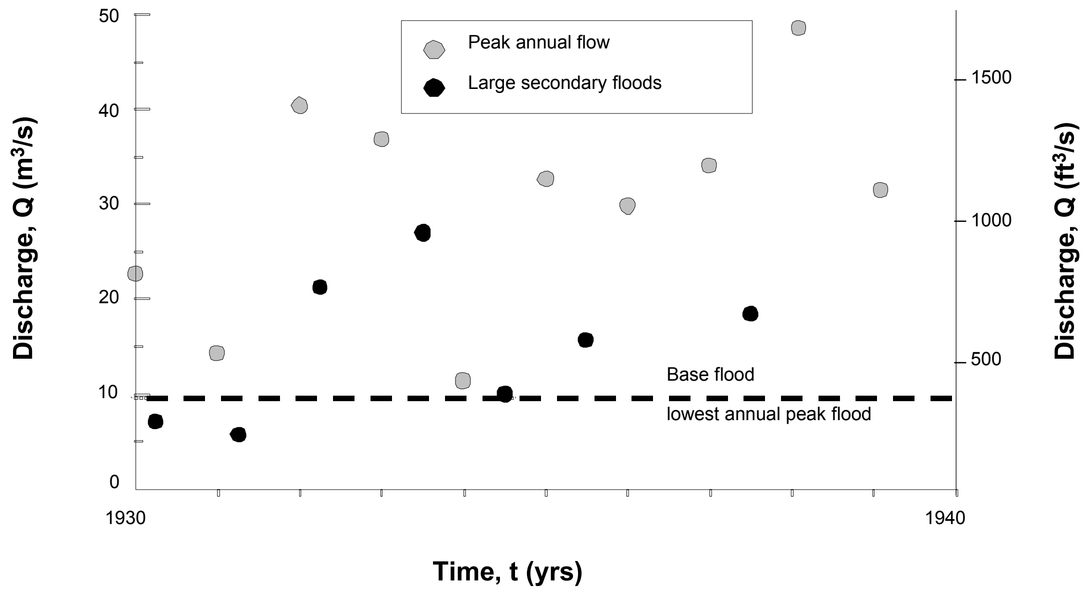

Figure 4.1. Peak annual and other large secondary flows, Mono Creek, CA

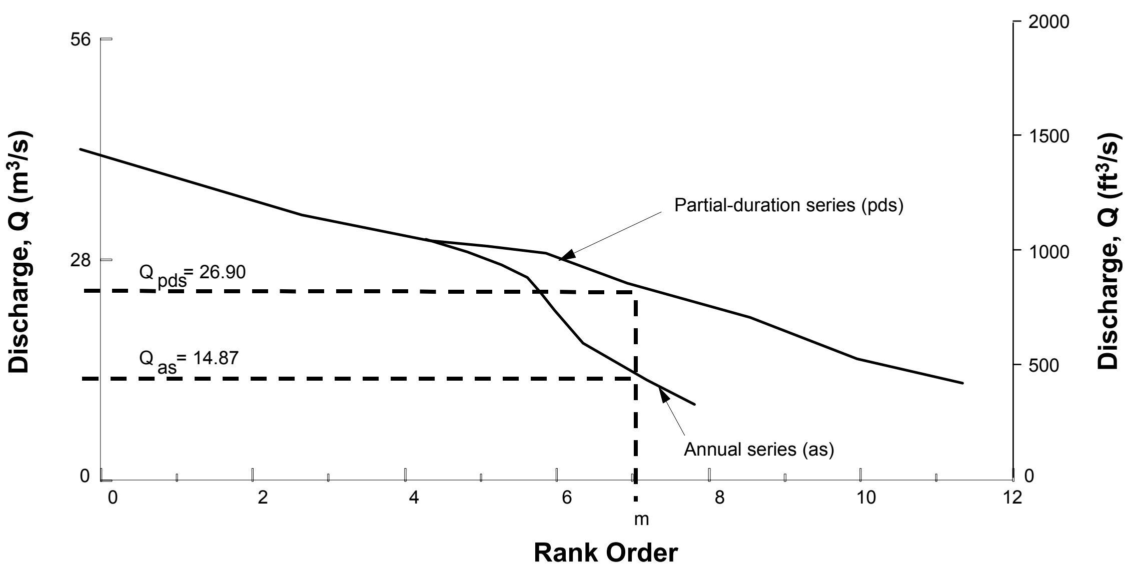

Figure 4.2. Annual and partial-duration series

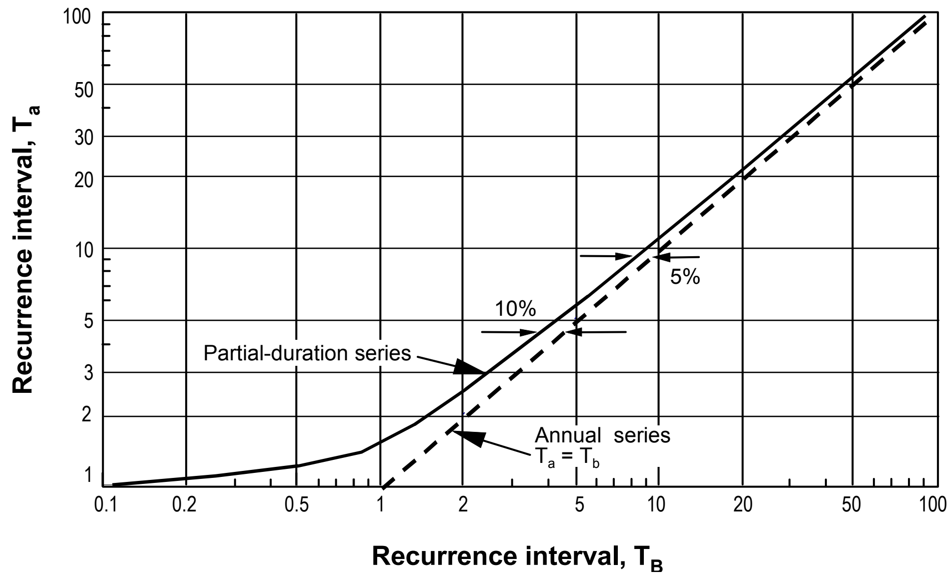

Figure 4.3. Relation between annual and partial-duration series

Table 4.2. Comparison of Annual and Partial-Duration Curves

Number of Years Flow is Exceeded per Hundred Years
(from Beard, 1962)

Annual-event	Partial-duration
1	1.00
2	2.02
5	5.10
10	10.50
20	22.30
30	35.60
40	51.00
50	69.30
60	91.70
63	100.00
70	120.00
80	161.00
90	230.00
95	300.00

4.2.2 Detection of Nonhomogeneity in the Annual Flood Series

Frequency analysis is a method based on order-theory statistics. Basic assumptions that should be evaluated prior to performing the analysis are:

The data are independent and identically distributed random events.

1. The data are from the sample population.
2. The data are assumed to be representative of the population.

3. The process generating these events is stationary with respect to time.

Obviously, using a frequency analysis assumes that no measurement or computational errors were made. When analyzing a set of data, the validity of the four assumptions can be statistically evaluated using tests such as the following:

- Runs test for randomness
- Mann-Whitney U test for homogeneity
- Kendall test for trend
- Spearman rank-order correlation coefficient for trend

The Kendall test is described by Hirsch, et al. (1982). The other tests are described in the British Flood Studies Report (National Environmental Research Council, 1975) and in the documentation for the Canadian flood-frequency program (Pilon and Harvey, 1992). A work group for revising USGS Bulletin 17B (1982) is currently writing a report that documents and illustrates these tests.

Another way to arrange data is according to their time of occurrence. Such an arrangement is called a time series. As an example of a time series, the same 29 years of data presented in Table 4.1 are arranged according to year of occurrence rather than magnitude and plotted in Figure 4.4.

This time series shows the temporal variation of the data and is an important step in data analysis. The analysis of time variations is called trend analysis and there are several methods that are used in trend analysis. The two most commonly used in hydrologic analysis are the moving-average method and the methods of curve fitting. A major difference between the moving-average method and curve fitting is that the moving-average method does not provide a mathematical equation for making estimates. It only provides a tabular or graphical summary from which a trend can be subjectively assessed. Curve fitting can provide an equation that can be used to make estimates. The various methods of curve fitting are discussed in more detail by Sanders (1980) and McCuen (1993).

The method of moving averages is presented here. Moving-average filtering reduces the effects of random variations. The method is based on the premise that the systematic component of a time series exhibits autocorrelation (i.e., correlation between nearby measurements) while the random fluctuations are not autocorrelated. Therefore, the averaging of adjacent measurements will eliminate the random fluctuations, with the result converging to a qualitative description of any systematic trend that is present in the data.

In general, the moving-average computation uses a weighted average of adjacent observations to produce a new time series that consists of the systematic trend. Given a time series Y_i, the filtered series $\hat{Y}_i$ is derived by:

$$\hat{Y}_i = \sum_{j=1}^{m} w_j Y_{i-k+j-1} \quad \text{for} \quad i = (k+1), (k+2), \ldots, (n-k) \qquad (4.2)$$

where,

m = the number of observations used to compute the filtered value (i.e., the smoothing interval)

w_j = the weight applied to value j of the series Y.

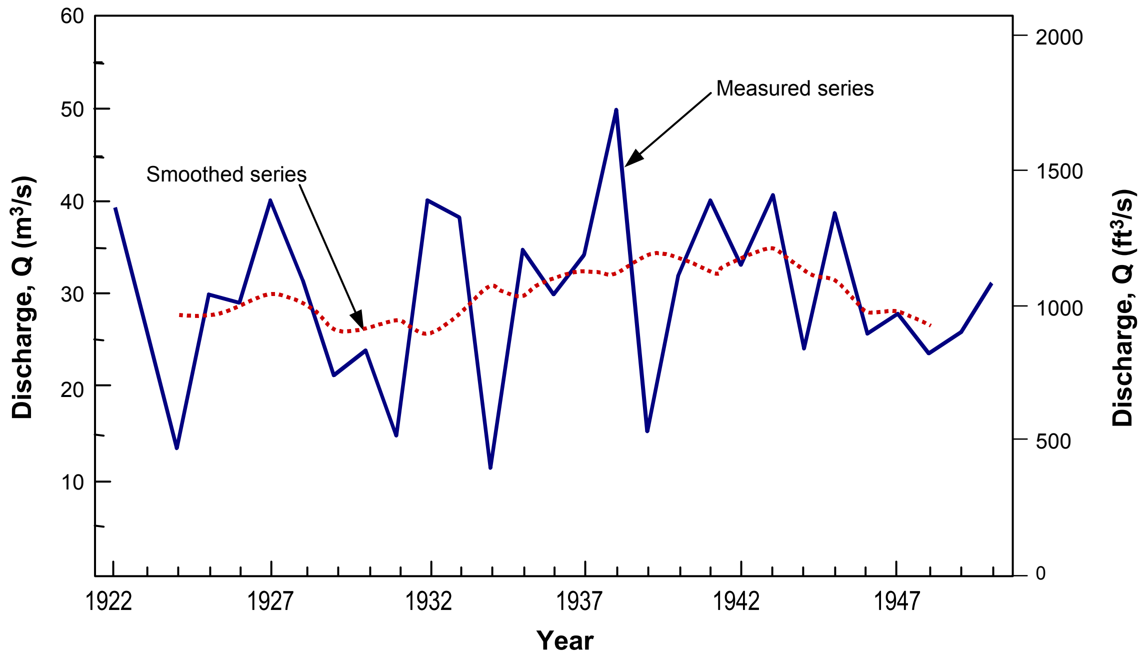

Figure 4.4. Measured and smoothed flood series for Mono Creek, CA

The smoothing interval should be an odd integer, with 0.5 (m-1) values of Y before observation i and 0.5 (m-1) values of Y after observation i is used to estimate the smoothed value $\hat{Y}$. A total of 2*k observations are lost; that is, while the length of the measured time series equals n, the smoothed series, $\hat{Y}$, has (n - 2k) values. The simplest weighting scheme would be the arithmetic mean (i.e., $w_j = 1/m$). Other weighting schemes give the greatest weight to the central point in the interval, with successively smaller weights given to points farther removed from the central point.

Moving-average filtering has several disadvantages. First, as described above, the approach loses 2*k observations, which may be a very limiting disadvantage for short record lengths. Second, a moving-average filter is not itself a mathematical representation, and thus forecasting with the filter is not possible; a structural form must still be calibrated to forecast any systematic trend identified by the filtering. Third, the choice of the smoothing interval is not always obvious, and it is often necessary to try several values in order to provide the best separation of systematic and random variation. Fourth, if the smoothing interval is not properly selected, it is possible to eliminate some of the systematic variation with the random variation.

A moving-average filter can be used to identify the presence of either a trend or a cycle. The smoothed series will enable the form of the trend or the period of the cycle to be estimated. A model can be developed to represent the systematic component and the model coefficients evaluated with a numerical fitting method.

Trend analysis plays an important role in evaluating the effects of changing land use and other time dependent parameters. Often through the use of trend analysis, future events can be estimated more rationally and past events are better understood.

Two examples will be used to demonstrate the use of moving-average smoothing. In both cases, a 5-year smoothing interval was used. Three-year intervals were not sufficient to clearly show the trend, and intervals longer than 5 years did not improve the ability to interpret the results.

<u>Example 4.1</u>. Table 4.1 contains the 29-year annual flood series for Mono Creek, CA; the series is shown in Figure 4.4. The calculated smoothed series is also listed in Table 4.1 and shown in Figure 4.4. The trend in the smoothed series is not hydrologically significant, which suggests that rainfall and watershed conditions have not caused a systematic trend during the period of record.

<u>Example 4.2</u>. Table 4.3 contains the 24-year annual flood series and smoothed series for Pond Creek, KY; the two series are shown in Figure 4.5. The Pond Creek watershed became urbanized in the late 1950s. Thus, the flood peaks tended to increase. This is evident from the obvious trend in the smoothed series during the period of urbanization. It appears that urbanization caused at least a doubling of flood magnitudes. While the smoothing does not provide a model of the effects of urbanization, the series does suggest the character of the effects of urbanization. Other possible causes of the trend should be investigated to provide some assurance that the urban development was the cause.

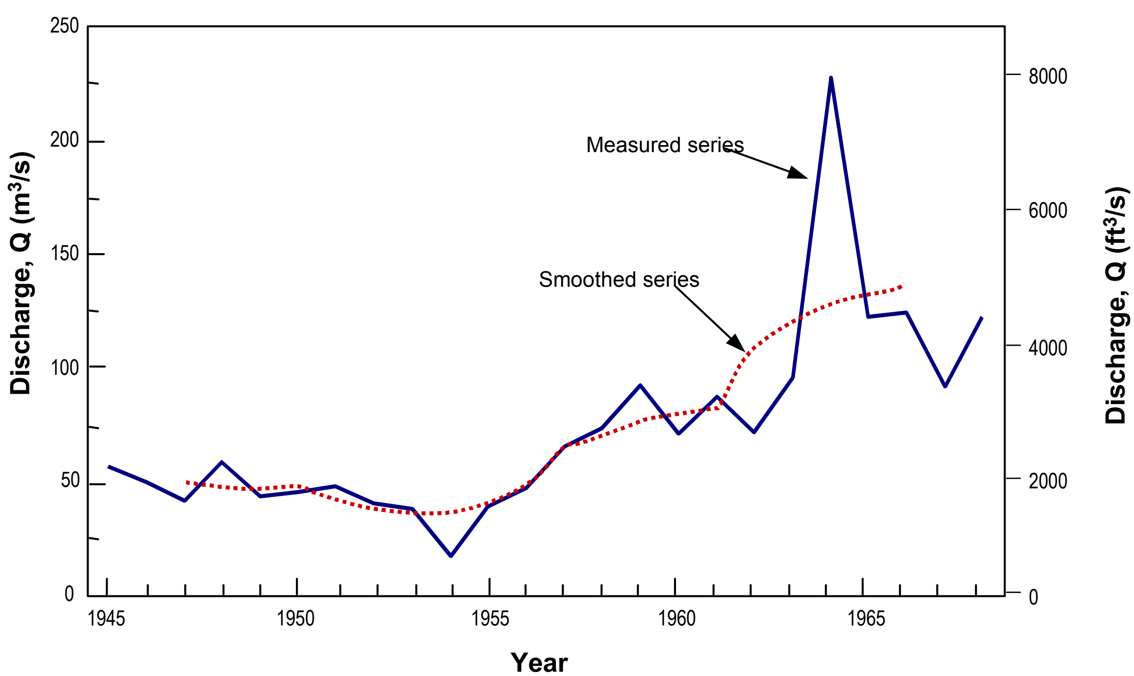

Figure 4.5. Measured and smoothed series for annual peak flows, Pond Creek, KY

Table 4.3. Computation of 5-year Moving Average of Peak Flows, Pond Creek, KY

Year	Annual Maximum (m^3/s)	Smoothed Series (m^3/s)	Annual Maximum (ft^3/s)	Smoothed Series (ft^3/s)
1945	56.7	-	2,002	-
1946	49.3	-	1,741	-
1947	41.4	49.8	1,462	1,760
1948	58.4	47.5	2,062	1,678
1949	43.4	47.2	1,532	1,668
1950	45.1	47.0	1,593	1,660
1951	47.9	42.8	1,691	1,513
1952	40.2	37.6	1,419	1,328
1953	37.7	36.4	1,331	1,286
1954	17.2	36.3	607	1,280
1955	39.1	41.2	1,381	1,454
1956	47.0	48.3	1,660	1,706
1957	64.9	63.4	2,292	2,237
1958	73.4	69.7	2,592	2,460
1959	92.4	77.7	3,263	2,744
1960	70.6	79.0	2,493	2,790
1961	87.3	83.4	3,083	2,944
1962	71.4	110.4	2,521	3,897
1963	95.2	120.7	3,362	4,261
1964	227.3	128.0	8,026	4,520
1965	122.1	132.0	4,311	4,661
1966	124.1	137.4	4,382	4,853
1967	91.3	-	3,224	-
1968	122.4	-	4,322	-

4.2.3 Arrangement by Geographic Location

The primary purpose of arranging flood data by geographic area is to develop a database for the analysis of peak flows at sites that are either ungaged or have insufficient data. Classically, flood data are grouped for basins with similar meteorologic and physiographic characteristics. Meteorologically, this means that floods are caused by storms with similar type rainfall intensities, durations, distributions, shapes, travel directions, and other climatic conditions. Similarity of physiographic features means that basin slopes, shapes, stream density, ground cover, geology, and hydrologic abstractions are similar among watersheds in the same region.

Some of these parameters are described quantitatively in a variety of ways while others are totally subjective. There can be considerable variation in estimates of watershed similarity in a geographical area. From a quantitative standpoint, it is preferable to consider the properties that describe the distribution of floods from different watersheds. These properties, which are described more fully in later parts of this section, include the variance, standard deviation, and coefficient of skew. Other methods can be used to test for hydrologic homogeneity such as the runoff per unit of drainage area, the ratio of various frequency floods to average floods, the standard error of estimate, and the residuals of regression analyses. The latter techniques are typical of those used to establish geographic areas for regional regression equations and other regional procedures for peak flow estimates.

4.2.4 Probability Concepts

The statistical analysis of repeated observations of an event (e.g., observations of peak annual flows) is based on the laws of probability. The probability of exceedence of a single peak flow, Q_A, is approximated by the relative number of exceedences of Q_A after a long series of observations, i.e.,

$$P_r(Q_A) = \frac{n_1}{n} = \frac{\text{No. of exceedences of some flood magnitude}}{\text{No. of observations (if large)}} \tag{4.3}$$

where,

n_1 = the frequency

n_1/n = relative frequency of Q_A.

Most people have an intuitive grasp of the concept of probability. They know that if a coin is tossed, there is an equal probability that a head or a tail will result. They know this because there are only two possible outcomes and that each is equally likely. Again, relying on past experience or intuition, when a fair die is tossed, there are six equally likely outcomes, any of the numbers 1, 2, 3, 4, 5, or 6. Each has a probability of occurrence of 1/6. So the chances that the number 3 will result from a single throw is 1 out of 6. This is fairly straightforward because all of the possible outcomes are known beforehand and the probabilities can be readily quantified.

On the other hand, the probability of a nonexceedence (or failure) of an event such as peak flow, Q_A, is given by:

$$P_r(\text{not } Q_A) = \frac{n - n_1}{n} = 1 - \frac{n_1}{n} = 1 - P_r(Q_A) \tag{4.4}$$

Combining Equations 4.3 and 4.4 yields:

$$P_r(Q_A) + P_r(\text{not } Q_A) = 1 \tag{4.5}$$

or the probability of an event being exceeded is between 0 and 1 (i.e., $0 \leq Pr(Q_A) \leq 1$). If an event is certain to occur, it has a probability of 1, and if it cannot occur at all, it has a probability of 0.

Given two independent flows, Q_A and Q_B, the probability of the successive exceedence of both Q_A and Q_B is given by:

$$P_r(Q_A \text{ and } Q_B) = P_r(Q_A)\, P_r(Q_B) \tag{4.6}$$

If the exceedence of a flow Q_A excludes the exceedence of another flow Q_2, the two events are said to be mutually exclusive. For mutually exclusive events, the probability of exceedence of either Q_A or Q_B is given by:

$$P_r(Q_A \text{ or } Q_B) = P_r(Q_A) + P_r(Q_B) \tag{4.7}$$

4.2.5 Return Period

If the exceedence probability of a given annual peak flow or its relative frequency determined from Equation 4.3 is 0.2, this means that there is a 20 percent chance that this flood, over a long period of time, will be exceeded in any one year. Stated another way, this flood will be exceeded on an average of once every 5 years. That time interval is called the return period, recurrence interval, or exceedence frequency.

The return period, T_r, is related to the probability of exceedence by:

$$T_r = \frac{1}{P_r(Q_A)} \tag{4.8}$$

The designer is cautioned to remember that a flood with a return period of 5 years does not mean this flood will occur once every 5 years. As noted, the flood has a 20 percent probability of being exceeded in any year, and there is no preclusion of the 5-year flood being exceeded in several consecutive years. Two 5-year floods can occur in two consecutive years; there is also a probability that a 5-year flood may not be exceeded in a 10-year period. The same is true for any flood of specified return period.

4.2.6 Estimation of Parameters

Flood frequency analysis uses sample information to fit a population, which is a probability distribution. These distributions have parameters that must be estimated in order to make probability statements about the likelihood of future flood magnitudes. A number of methods for estimating the parameters are available. USGS Bulletin 17B (1982) uses the method of moments, which is just one of the parameter-estimation methods. The method of maximum likelihood is a second method.

The method of moments equates the moments of the sample flood record to the moments of the population distribution, which yields equations for estimating the parameters of the population as a function of the sample moments. As an example, if the population is assumed to follow distribution f(x), then the sample mean ($\overline{X}$) could be related to the definition of the population mean (μ):

$$\overline{X} = \int_{-\infty}^{\infty} x f(x) dx \tag{4.9}$$

and the sample variance (S^2) could be related to the definition of the population variance (σ^2):

$$S^2 = \int_{-\infty}^{\infty} (X - \mu)^2 f(x) dx \tag{4.10}$$

Since f(x) is a function that includes the parameters (μ and σ^2), the solution of Equations 4.9 and 4.10 will be expressions that relate $\overline{X}$ and S^2 to the parameters μ and σ^2.

While maximum likelihood estimation (MLE) is not used in USGS Bulletin 17B (1982) and it is more involved than the method of moments, it is instructive to put MLE in perspective. MLE defines a likelihood function that expresses the probability of obtaining the population parameters given that the measured flood record has occurred. For example, if μ and σ are the population parameters and the flood record X contains N events, the likelihood function is:

$$L(\mu,\sigma \mid X_1, X_2, \ldots, X_N) = \prod_{i=1}^{N} f(X_i \mid \mu,\sigma) \tag{4.11}$$

where $f(X_I \mid \mu, \sigma)$ is the probability distribution of X as a function of the parameters. The solution of Equation 4.11 will yield expressions for estimating μ and σ from the flood record X.

4.2.7 Frequency Analysis Concepts

Future floods cannot be predicted with certainty. Therefore, their magnitude and frequency are treated using probability concepts. To do this, a sample of flood magnitudes are obtained and analyzed for the purpose of estimating a population that can be used to represent flooding at that location. The assumed population is then used in making projections of the magnitude and frequency of floods. It is important to recognize that the population is estimated from sample information and that the assumed population, not the sample, is then used for making statements about the likelihood of future flooding. The purpose of this section is to introduce concepts that are important in analyzing sample flood data in order to identify a probability distribution that can represent the occurrence of flooding.

4.2.7.1 Frequency Histograms

Frequency distributions are used to facilitate an analysis of sample data. A frequency distribution, which is sometimes presented as a histogram, is an arrangement of data by classes or categories with associated frequencies of each class. The frequency distribution shows the magnitude of past events for certain ranges of the variable. Sample probabilities can also be computed by dividing the frequencies of each interval by the sample size.

A frequency distribution or histogram is constructed by first examining the range of magnitudes (i.e., the difference between the largest and the smallest floods) and dividing this range into a number of conveniently sized groups, usually between 5 and 20. These groups are called class intervals. The size of the class interval is simply the range divided by the number of class intervals selected. There is no precise rule concerning the number of class intervals to select, but the following guidelines may be helpful:

1. The class intervals should not overlap, and there should be no gaps between the bounds of the intervals.

2. The number of class intervals should be chosen so that most class intervals have at least one event.

3. It is preferable that the class intervals are of equal width.

4. It is also preferable for most class intervals to have at least five occurrences; this may not be practical for the first and last intervals.

Example 4.3. Using these rules, the discharges for Mono Creek listed in Table 4.1 are placed into a frequency histogram using class intervals of 5 m^3/s (SI) and 200 ft^3/s (CU units) (see Table 4.4). These data can also be represented graphically by a frequency histogram as shown in Figure 4.6. Since relative frequency has been defined as the number of events in a certain class of events divided by the sample size, the histogram can also represent relative frequency (or probability) as shown on the right-hand ordinate of Figure 4.6.

From this frequency histogram, several features of the data can now be illustrated. Notice that there are some ranges of magnitudes that have occurred more frequently than others; also notice that the data are somewhat spread out and that the distribution of the ordinates is not symmetrical. While an effort was made to have frequencies of five or more, this was not possible with the class intervals selected. Because of the small sample size, it is difficult to assess the distribution of the population using the frequency histogram. It should also be noted that because the CU unit intervals are not a conversion from the SI, they represent an alternative interval selection. This illustrates that interval selection may influence the appearance of a histogram.

Table 4.4. Frequency Histogram and Relative Frequency Analysis of Annual Flood Data for Mono Creek

(a) 5 m^3/s intervals (SI)

Interval of Annual Floods (m^3/s)	Frequency	Relative Frequency	Cumulative Frequency
0 – 9.99	0	0.000	0.000
10 – 14.99	3	0.104	0.104
15 – 19.99	1	0.034	0.138
20 – 24.99	4	0.138	0.276
25 – 29.99	5	0.172	0.448
30 – 34.99	8	0.276	0.724
35 – 39.99	3	0.104	0.828
40 – 44.99	4	0.138	0.966
45 or larger	1	0.034	1.000

(b) 200 ft^3/s intervals (CU Units)

Interval of Annual Floods (ft^3/s)	Frequency	Relative Frequency	Cumulative Frequency
0 – 199	0	0.000	0.000
200 – 399	0	0.000	0.000
400 – 599	4	0.138	0.138
600 – 799	1	0.034	0.172
800 – 999	7	0.241	0.414
1000 – 1199	7	0.241	0.655
1200 – 1399	5	0.172	0.828
1400 – 1599	4	0.138	0.966
1600 – 1799	1	0.034	1.000

Example 4.4. Many flood records have relatively small record lengths. For such records, histograms may not be adequate to assess the shape characteristics of the distribution of floods. The flood record for Pond Creek of Table 4.3 provides a good illustration. With a record length of 24, it would be impractical to use more than 5 or 6 intervals when creating a histogram. Three histograms were compiled from the annual flood series (see Table 4.5). The first histogram uses an interval of 40 m^3/s (1,412 ft^3 /s) and results in a hydrograph-like shape, with few values in the lowest cell and a noticeable peak in the second cell. The second histogram uses an interval of 50 m^3/s (1,766 ft^3/s). This produces a box-like shape with the first two cells having a large number of occurrences and the other cells very few, with one intermediate cell not having any occurrences. The third histogram uses an unequal cell width and produces an exponential-decay shape. These results indicate that short record lengths make it difficult to identify the distribution of floods.

Table 4.5. Alternative Frequency (f) Histograms of the Pond Creek, KY, Annual Maximum Flood Record (1945-1968)

Interval	Histogram 1 Frequency	Histogram 2 Frequency	Histogram 3 Frequency	**Histogram 3 Interval**	
				(m^3/s)	**(ft^3/s)**
1	3	10	10	0 – 50	0 – 1,765
2	13	10	5	50 – 75	1,766 – 2,648
3	4	3	5	75 – 100	2,649 – 3,531
4	3	0	3	100 – 150	3,532 – 5,297
5	1	1	1	> 150	> 5,297

4.2.7.2 Central Tendency

The clustering of the data about particular magnitudes is known as central tendency, of which there are a number of measures. The most frequently used is the average or the mean value. The mean value is calculated by summing all of the individual values of the data and dividing the total by the number of individual data values

$$\overline{Q} = \frac{\sum_{i=1}^{n} Q_i}{n} \tag{4.12}$$

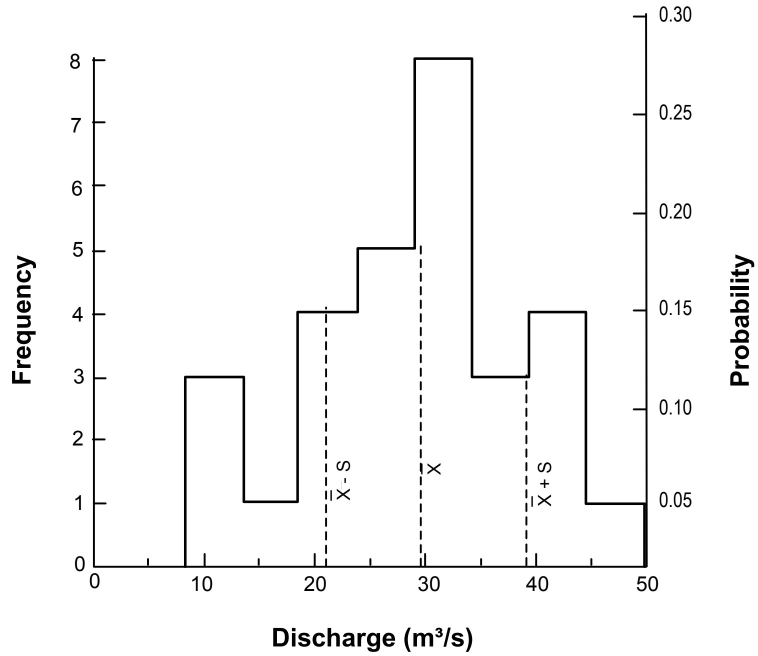

Figure 4.6a. Sample frequency histogram and probability, Mono Creek, CA ($\bar{X}$ = 30.0 m^3/s and S = 9.3 m^3/s)

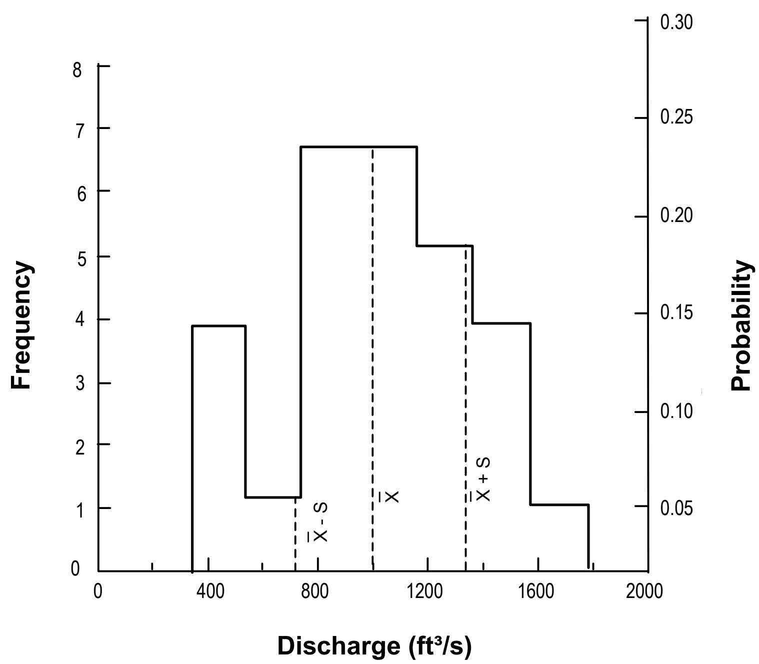

Figure 4.6b. Sample frequency histogram and probability, Mono Creek, CA ($\bar{X}$ = 1060 ft^3/s and S = 330 ft^3/s)

where,

$\overline{Q}$ = average or mean peak.

The median, another measure of central tendency, is the value of the middle item when the items are arranged according to magnitude. When there is an even number of items, the median is taken as the average of the two central values.

The mode is a third measure of central tendency. The mode is the most frequent or most common value that occurs in a set of data. For continuous variables, such as discharge rates, the mode is defined as the central value of the most frequent class interval.

4.2.7.3 Variability

The spread of the data is called dispersion. The most commonly used measure of dispersion is the standard deviation. The standard deviation, S, is defined as the square root of the mean square of the deviations from the average value. This is shown symbolically as:

$$S = \left[\frac{\sum_{i=1}^{n} (Q_i - \overline{Q})^2}{n - 1} \right]^{0.5} = \overline{Q} \left[\frac{\sum_{i=1}^{n} \left(\frac{Q_i}{\overline{Q}} - 1 \right)^2}{n - 1} \right]^{0.5} \tag{4.13}$$

The second expression on the right-hand side of Equation 4.13 is often used to facilitate and improve on the accuracy of hand calculations.

Another measure of dispersion of the flood data is the variance, or simply the standard deviation squared. A measure of relative dispersion is the coefficient of variation, V, or the standard deviation divided by the mean peak:

$$V = \frac{S}{\overline{Q}} \tag{4.14}$$

4.2.7.4 Skew

The symmetry of the frequency distribution, or more accurately the asymmetry, is called skew. One common measure of skew is the coefficient of skew, G. The skew coefficient is calculated by:

$$G = \frac{n \sum_{i=1}^{n} (Q_i - \overline{Q})^3}{(n - 1)(n - 2) S^3} = \frac{n \sum_{i=1}^{n} \left(\frac{Q_i}{\overline{Q}} - 1 \right)^3}{(n - 1)(n - 2) V^3} \tag{4.15}$$

where all symbols are as previously defined. Again, the second expression on the right-hand side of the equation is for ease of hand computations.

If a frequency distribution is perfectly symmetrical, the coefficient of skew is zero. If the distribution has a longer "tail" to the right of the central maximum than to the left, the distribution has a positive skew and G would be positive. If the longer tail is to the left of the central maximum, the distribution has a negative coefficient of skew.

Example 4.5. The computations below illustrate the computation of measures of central tendency, standard deviation, variance, and coefficient of skew for the Mono Creek frequency distribution shown in Figure 4.6 based on the data provided in Table 4.6. The mean value of the sample of floods is 30 m^3/s (1,060 ft^3/s), the standard deviation is 9.3 m^3/s (330 ft^3/s), and the coefficient of variation is 0.31. The coefficient of skew is –0.19, which indicates that the distribution is skewed negatively to the left. For the flow data in Table 4.6, the median value is 30.0 m^3/s (1,060 ft^3/s). Computed values of the mean and standard deviation are also identified in Figure 4.6.

Variable	Value in SI	Value in CU
$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$	$\frac{868.6}{29} = 30.0 \ m^3/s$	$\frac{3066}{29} = 1058 \ ft^3/s$
$S = \overline{X}\left[\frac{\sum_{i=1}^{n}\left(\frac{X_i}{\overline{X}} - 1\right)^2}{n-1}\right]^{0.5}$	$30.0\left[\frac{2.677}{28}\right]^{0.5} = 9.3 \ m^3/s$	$1058\left[\frac{2.677}{28}\right]^{0.5} = 327 \ ft^3/s$
$V = \frac{S}{\overline{X}}$	$\frac{9.3}{30.0} = 0.31$	$\frac{327}{1{,}058} = 0.31$
$G = \frac{n\sum_{i=1}^{n}\left(\frac{X_i}{\overline{X}} - 1\right)^3}{(n-1)(n-2)V^3}$	$\frac{29(-0.1448)}{28(27)(0.31)^3} = -0.19$	$\frac{29(-0.1448)}{28(27)(0.31)^3} = -0.19$

Table 4.6. Computation of Statistical Characteristics: Annual Maximum Flows for Mono Creek, CA

Year	Rank	Annual Maximum (m^3/s)	Annual Maximum (ft^3/s)	$[(X/\bar{X})]$	$[(X/\bar{X})-1]$	$[(X/\bar{X})-1]^2$	$[(X/\bar{X})-1]^3$
1938	1	49.8	1,760	1.664	0.664	0.441	0.2929
1943	2	40.8	1,440	1.362	0.362	0.131	0.0473
1927	3	40.2	1,420	1.343	0.343	0.117	0.0402
1932	4	40.2	1,420	1.343	0.343	0.117	0.0402
1941	5	40.2	1,420	1.343	0.343	0.117	0.0402
1922	6	39.4	1,390	1.314	0.314	0.099	0.0310
1945	7	38.8	1,370	1.295	0.295	0.087	0.0257
1933	8	38.2	1,350	1.276	0.276	0.076	0.0211
1935	9	34.8	1,230	1.163	0.163	0.027	0.0043
1937	10	34.3	1,210	1.144	0.144	0.021	0.0030
1942	11	33.1	1,170	1.106	0.106	0.011	0.0012
1940	12	32.0	1,130	1.068	0.068	0.005	0.0003
1928	13	31.4	1,110	1.049	0.049	0.002	0.0001
1950	14	31.2	1,100	1.040	0.040	0.002	0.0001
1925	15	30.0	1,060	1.002	0.002	0.000	0.0000
1936	16	30.0	1,060	1.002	0.002	0.000	0.0000
1926	17	29.2	1,030	0.974	-0.026	0.001	0.0000
1947	18	28.0	988	0.934	-0.066	0.004	-0.0003
1923	19	26.6	940	0.889	-0.111	0.012	-0.0014
1949	20	25.9	916	0.866	-0.134	0.018	-0.0024
1946	21	25.8	910	0.860	-0.140	0.019	-0.0027
1944	22	24.2	855	0.808	-0.192	0.037	-0.0070
1930	23	24.0	848	0.802	-0.198	0.039	-0.0078
1948	24	23.7	838	0.792	-0.208	0.043	-0.0090
1929	25	21.2	750	0.709	-0.291	0.085	-0.0246
1939	26	15.3	540	0.511	-0.489	0.240	-0.1173
1931	27	14.9	525	0.496	-0.504	0.254	-0.1277
1924	28	13.8	488	0.461	-0.539	0.290	-0.1562
1934	29	11.4	404	0.382	-0.618	0.382	-0.2361
	TOTAL	**868.4**	**30,672**			**2.677**	**-0.1449**

4.2.7.5 Generalized and Weighted Skew

Three methods are available for representing the skew coefficient. These include the station skew, a generalized skew, and a weighted skew. Since the skew coefficient is very sensitive to extreme values, the station skew (i.e., the skew coefficient computed from the actual data) may not be accurate if the sample size is small. In this case, USGS Bulletin 17B (1982) recommends use of a generalized skew coefficient determined from a map that shows isolines of generalized skew coefficients of the logarithms of annual maximum stream flows throughout the United States. A map of generalized skew is provided in Bulletin 17B. This map also gives average skew coefficients by one-degree quadrangles over most of the country.

Often the station skew and generalized skew can be combined to provide a better estimate for a given sample of flood data. USGS Bulletin 17B (1982) outlines a procedure based on the concept that the mean-square error (MSE) of the weighted estimate is minimized by weighting the station and generalized skews in inverse proportion to their individual MSEs, which are defined as the sum of the squared differences between the true and estimated values of a quantity divided by the number of observations. In analytical form, this concept is given by the equation:

$$G_W = \frac{MSE_{\overline{G}}\,(G) + MSE_G\,(\overline{G})}{MSE_{\overline{G}} + MSE_G} \tag{4.16}$$

where,

G_W = weighted skew
G = station skew
$\overline{G}$ = generalized skew
MSE_G, $MSE_{\overline{G}}$ = mean-square errors for the station and generalized skews, respectively.

Equation 4.16 is based on the assumption that station and generalized skew are independent. If they are independent, the weighted estimate will have a lower variance than either the station or generalized skew.

When $\overline{G}$ is taken from the map of generalized skews in USGS Bulletin 17B (1982), $MSE_{\overline{G}}$ = 0.302. The value of MSE_G can be obtained from Table 4.7, which is from Bulletin 17B, or approximated by the equation:

$$MSE_G = 10^{\left(A - B\left[\log_{10}\left(n/10\right)\right]\right)} \tag{4.17a}$$

where n is the record length and

$$A = -0.33 + 0.08|G| \quad \text{for } |G| \le 0.90 \tag{4.17b}$$

$$A = -0.52 + 0.30|G| \quad \text{for } |G| > 0.90 \tag{4.17c}$$

and

$$B = 0.94 - 0.26|G| \quad \text{for } |G| \le 1.50 \tag{4.17d}$$

$$B = 0.55 \quad \text{for } |G| > 1.50 \tag{4.17e}$$

If the difference between the generalized and station skews is greater than 0.5, the data and basin characteristics should be reviewed, possibly giving more weight to the station skew.

Table 4.7. Summary of Mean Square Error of Station Skew a Function of Record Length and Station Skew

	Record Length, N or H (years)									
Skew	10	20	30	40	50	60	70	80	90	100
0.0	0.468	0.244	0.167	0.127	0.103	0.087	0.075	0.066	0.059	0.054
0.1	0.476	0.253	0.175	0.134	0.109	0.093	0.080	0.071	0.064	0.058
0.2	0.485	0.262	0.183	0.142	0.116	0.099	0.086	0.077	0.069	0.063
0.3	0.494	0.272	0.192	0.150	0.123	0.105	0.092	0.082	0.074	0.068
0.4	0.504	0.282	0.201	0.158	0.131	0.113	0.099	0.089	0.080	0.073
0.5	0.513	0.293	0.211	0.167	0.139	0.120	0.106	0.095	0.087	0.079
0.6	0.522	0.303	0.221	0.176	0.148	0.128	0.114	0.102	0.093	0.086
0.7	0.532	0.315	0.231	0.186	0.157	0.137	0.122	0.110	0.101	0.093
0.8	0.542	0.326	0.243	0.196	0.167	0.146	0.130	0.118	0.109	0.100
0.9	0.562	0.345	0.259	0.211	0.181	0.159	0.142	0.130	0.119	0.111
1.0	0.603	0.376	0.285	0.235	0.202	0.178	0.160	0.147	0.135	0.126
1.1	0.646	0.410	0.315	0.261	0.225	0.200	0.181	0.166	0.153	0.143
1.2	0.692	0.448	0.347	0.290	0.252	0.225	0.204	0.187	0.174	0.163
1.3	0.741	0.488	0.383	0.322	0.281	0.252	0.230	0.212	0.197	0.185
1.4	0.794	0.533	0.422	0.357	0.314	0.283	0.259	0.240	0.224	0.211
1.5	0.851	0.581	0.465	0.397	0.351	0.318	0.292	0.271	0.254	0.240
1.6	0.912	0.623	0.498	0.425	0.376	0.340	0.313	0.291	0.272	0.257
1.7	0.977	0.667	0.534	0.456	0.403	0.365	0.335	0.311	0.292	0.275
1.8	1.047	0.715	0.572	0.489	0.432	0.391	0.359	0.334	0.313	0.295
1.9	1.122	0.766	0.613	0.523	0.463	0.419	0.385	0.358	0.335	0.316
2.0	1.202	0.821	0.657	0.561	0.496	0.449	0.412	0.383	0.359	0.339
2.1	1.288	0.880	0.704	0.601	0.532	0.481	0.442	0.410	0.385	0.363
2.2	1.380	0.943	0.754	0.644	0.570	0.515	0.473	0.440	0.412	0.389
2.3	1.479	1.010	0.808	0.690	0.610	0.552	0.507	0.471	0.442	0.417
2.4	1.585	1.083	0.866	0.739	0.654	0.592	0.543	0.505	0.473	0.447
2.5	1.698	1.160	0.928	0.792	0.701	0.634	0.582	0.541	0.507	0.479
2.6	1.820	1.243	0.994	0.849	0.751	0.679	0.624	0.580	0.543	0.513
2.7	1.950	1.332	1.066	0.910	0.805	0.728	0.669	0.621	0.582	0.550
2.8	2.089	1.427	1.142	0.975	0.862	0.780	0.716	0.666	0.624	0.589
2.9	2.239	1.529	1.223	1.044	0.924	0.836	0.768	0.713	0.669	0.631
3.0	2.399	1.638	1.311	1.119	0.990	0.895	0.823	0.764	0.716	0.676

4.2.8 Probability Distribution Functions

If the frequency histogram from a very large population of floods was constructed, it would be possible to define very small class intervals and still have a number of events in each interval. Under these conditions, the frequency histogram would approach a smooth curve (see Figure 4.7) where the ordinate axis density units are the inverse of the abscissa units. This curve, which is called the probability density function, f(Q), encloses an area of 1.0 or:

$$\int_{-\infty}^{\infty} f(Q)dQ = 1 \tag{4.18}$$

The cumulative distribution function, F(Q), equals the area under the probability density function, f(Q), from $-\infty$ to Q:

$$F(Q) = \int_{\infty}^{Q} f(Q)dQ \tag{4.18a}$$

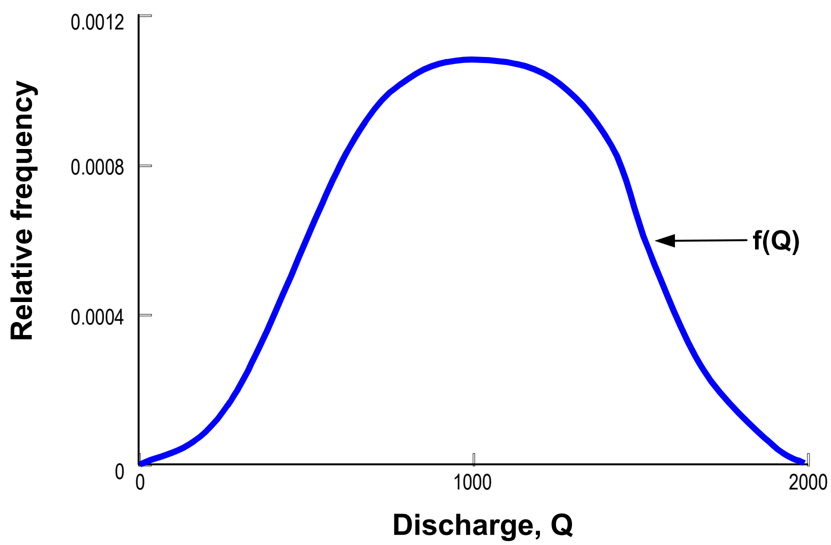

Figure 4.7. Probability density function

Equation 4.18 is a mathematical statement that the sum of the probabilities of all events is equal to unity. Two conditions of hydrologic probability are readily illustrated from Equations 4.18 and 4.18a. Figure 4.8a shows that the probability of a flow Q falling between two known flows, Q_1 and Q_2, is the area under the probability density curve between Q_1 and Q_2. Figure 4.8b shows the probability that a flood Q exceeds Q_1 is the area under the curve from Q_1 to infinity. From Equation 4.18a, this probability is given by $F(Q > Q_1) = 1 - F(Q < Q_1)$.

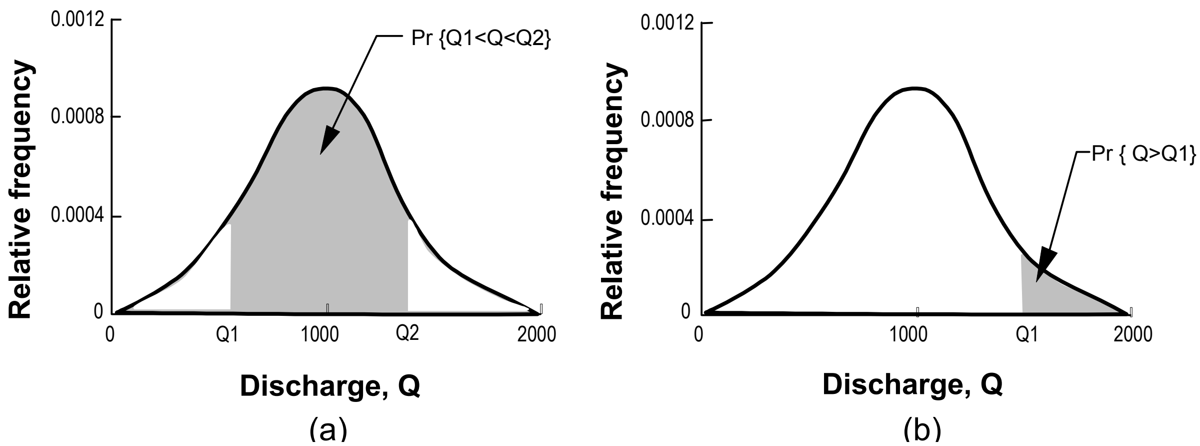

Figure 4.8. Hydrologic probability from density functions

As can be seen from Figure 4.8, the calculation for probability from the density function is somewhat tedious. A further refinement of the frequency distribution is the cumulative frequency distribution. Table 4.4 illustrates the development of a cumulative frequency distribution, which is simply the cumulative total of the relative frequencies by class interval. For each range of flows, Table 4.4 defines the number of times that floods equal or exceed the lower limit of the class interval and gives the cumulative frequency.

Using the cumulative frequency distribution, it is possible to compute directly the nonexceedence probability for a given magnitude. The nonexceedence probability is defined as the probability that the specified value will not be exceeded. The exceedence probability is 1.0 minus the nonexceedence probability. The sample cumulative frequency histogram for the Mono Creek, CA, annual flood series is shown in Figure 4.9.

Again, if the sample were very large so that small class intervals could be defined, the histogram becomes a smooth curve that is defined as the cumulative probability function, F(Q), shown in Figure 4.10a. This figure shows the area under the curve to the left of each Q of Figure 4.7 and defines the probability that the flow will be less than some stated value (i.e., the nonexceedence probability).

Another convenient representation for hydrologic analysis is the complementary probability function, G(Q), defined as:

$$G(Q) = 1 - F(Q) = P_r(Q \geq Q_1) \tag{4.19}$$

The function, G(Q), shown in Figure 4.10b, is the exceedence probability (i.e., the probability that a flow of a given magnitude will be equaled or exceeded).

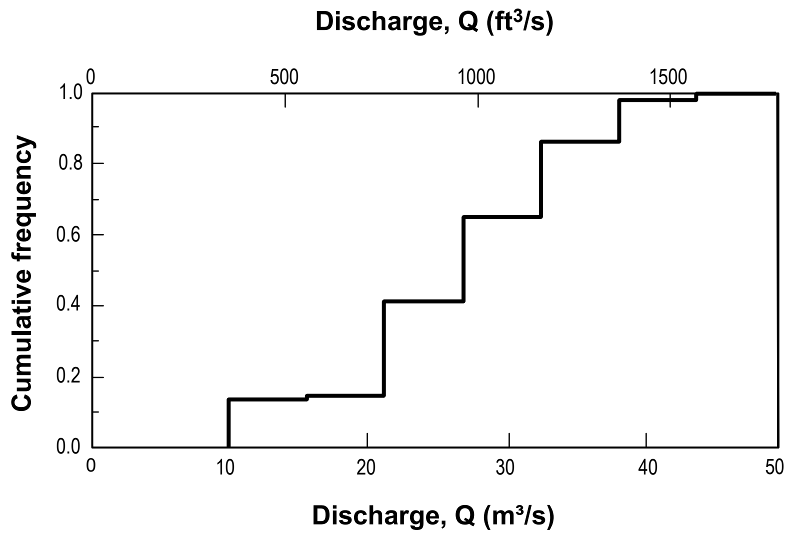

Figure 4.9. Cumulative frequency histogram, Mono Creek, CA

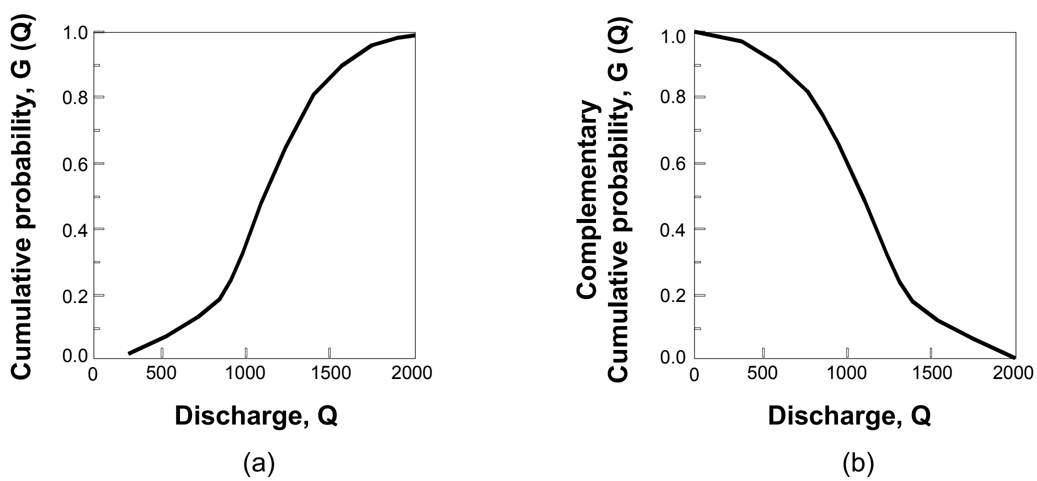

Figure 4.10. Cumulative and complementary cumulative distribution functions

4.2.9 Plotting Position Formulas

When making a flood frequency analysis, it is common to plot both the assumed population and the peak discharges of the sample. To plot the sample values on frequency paper, it is necessary to assign an exceedence probability to each magnitude. A plotting position formula is used for this purpose.

A number of different formulas have been proposed for computing plotting position probabilities, with no unanimity on the preferred method. Beard (1962) illustrates the nature of this problem. If a very long period of record, say 2,000 years, is broken up into 100 20-year records and each is analyzed separately, then the highest flood in each of these 20-year records will have the same probability of occurrence of 0.05. Actually, one of these 100 highest floods is the 1 in 2,000-year flood, which is a flood with an exceedence probability of 0.0005. Some of the records will also contain 100-year floods and many will contain floods in excess of the true 20-year flood. Similarly some of the 20-year records will contain highest floods that are less than the true 20-year flood.

A general formula for computing plotting positions is:

$$P = \frac{i - a}{(n - a - b + 1)} \qquad (4.20)$$

where,

i = rank order of the ordered flood magnitudes, with the largest flood having a rank of 1
n = record length
a, b = constants for a particular plotting position formula.

The Weibull, P_w (a = b = 0), Hazen, P_h (a = b = 0.5), and Cunnane, P_c (a = b = 0.4) are three possible plotting position formulas:

$$P_w = \frac{i}{n + 1} \qquad (4.21a)$$

$$P_h = \frac{i - 0.5}{n} \qquad (4.21b)$$

$$P_c = \frac{i - 0.4}{n + 0.2} \qquad (4.21c)$$

The data are plotted by placing a point for each value of the flood series at the intersection of the flood magnitude and the exceedence probability computed with the plotting position formula. The plotted data should approximate the population line if the assumed population model is a reasonable assumption.

For the partial-duration series where the number of floods exceeds the number of years of record, Beard (1962) recommends:

$$P = \frac{2i - 1}{2n} = \frac{i - 0.5}{n} \qquad (4.22)$$

where i is the rank order number of the event and n is the record length.

4.3 STANDARD FREQUENCY DISTRIBUTIONS

Several cumulative frequency distributions are commonly used in the analysis of hydrologic data and, as a result, they have been studied extensively and are now standardized. The frequency distributions that have been found most useful in hydrologic data analysis are the normal distribution, the log-normal distribution, the Gumbel extreme value distribution, and the log-Pearson Type III distribution. The characteristics and application of each of these distributions will be presented in the following sections.

4.3.1 Normal Distribution

The normal or Gaussian distribution is a classical mathematical distribution commonly used in the analysis of natural phenomena. The normal distribution has a symmetrical, unbounded, bell-shaped curve with the maximum value at the central point and extending from $-\infty$ to $+\infty$. The normal distribution is shown in Figure 4.11a.

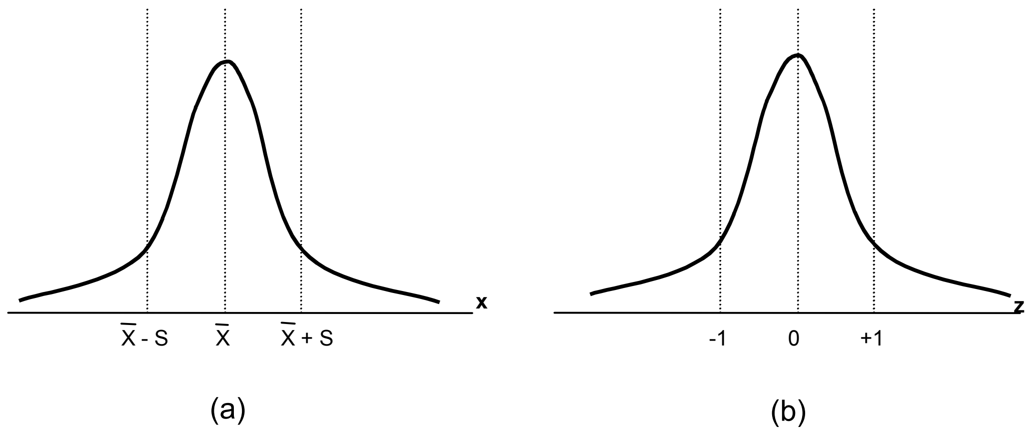

Figure 4.11. (a) Normal probability distribution; (b) Standard normal distribution

For the normal distribution, the maximum value occurs at the mean. Because of symmetry, half of the flows will be below the mean and half are above. Another characteristic of the normal distribution curve is that 68.3 percent of the events fall between ±1 standard deviation (S), 95 percent of the events fall within ±2S, and 99.7 percent fall within ±3S. In a sample of flows, these percentages will be approximated.

For the normal distribution, the coefficient of skew is zero. The function describing the normal distribution curve is:

$$f(X) = \frac{e^{-\left[(x-\overline{x})^2/2S^2\right]}}{S\sqrt{2\pi}} \tag{4.23}$$

Note that only two parameters are necessary to describe the normal distribution: the mean value, $\overline{X}$, and the standard deviation, S.

One disadvantage of the normal distribution is that it is unbounded in the negative direction whereas most hydrologic variables are bounded and can never be less than zero. For this reason and the fact that many hydrologic variables exhibit a pronounced skew, the normal distribution usually has limited applications. However, these problems can sometimes be overcome by performing a log transform on the data. Often the logarithms of hydrologic variables are normally distributed.

4.3.1.1 Standard Normal Distribution

A special case of the normal distribution of Equation 4.23 is called the standard normal distribution and is represented by the variate z (see Figure 4.11b). The standard normal distribution always has a mean of 0 and a standard deviation of 1. If the random variable X has a normal distribution with mean $\overline{X}$ and standard deviation S, values of X can be transformed so that they have a standard normal distribution using the following transformation:

$$z = \frac{X - \overline{X}}{S} \tag{4.24}$$

If $\overline{X}$, S, and z for a given frequency are known, then the value of X corresponding to the frequency can be computed by algebraic manipulation of Equation 4.24:

$$X = \overline{X} + zS \tag{4.25}$$

To illustrate, the 10-year event has an exceedence probability of 0.10 or a nonexceedence probability of 0.90. Thus, the corresponding value of z from Table 4.8 is 1.2816. If floods have a normal distribution with a mean of 120 m^3/s (4,240 ft^3/s) and a standard deviation of 35 m^3/s (1,230 ft^3/s), the 10-year flood for a normal distribution is computed with Equation 4.25:

Variable	Value in SI	Value in CU
$X = \overline{X} + zS$	$= 120 + 1.2816(35) = 165\ m^3/s$	$= 4240 + 1.2816(1230) = 165\ ft^3/s$

Similarly, the frequency of a flood of 181 m^3/s (6,390 ft^3/s) can be estimated using the transform of Equation 4.24:

Variable	Value in SI	Value in CU
$z = \frac{X - \overline{X}}{S}$	$= \frac{181 - 120}{35} = 1.75$	$= \frac{6390 - 4240}{1230} = 1.75$

From Table 4.8, this corresponds to an exceedence probability of 4 percent, which is the 25-year flood.

Table 4.8. Selected Values of the Standard Normal Deviate (z) for the Cumulative Normal Distribution

Exceedence Probability %	Return Period (yrs)	z
50	2	0.0000
20	5	0.8416
10	10	1.2816
4	25	1.7507
2	50	2.0538
1	100	2.3264
0.2	500	2.8782

4.3.1.2 Frequency Analysis for a Normal Distribution

An arithmetic-probability graph has a specially transformed horizontal probability scale. The horizontal scale is transformed in such a way that the cumulative distribution function for data that follow a normal distribution will plot as a straight line. If a series of peak flows that are normally distributed are plotted against the cumulative frequency function or the exceedence frequency on the probability scale, the data will plot as a straight line with the equation:

$$X = \overline{X} + K\,S \qquad (4.26)$$

where X is the flood flow at a specified frequency. The value of K is the frequency factor of the distribution. For the normal distribution, K equals z where z is taken from Table 4.8.

The procedure for developing a frequency curve for the normal distribution is as follows:

1. Compute the mean $\overline{X}$ and standard deviation S of the annual flood series.

2. Plot two points on the probability paper: (a) $\overline{X}$ + S at an exceedence probability of 0.159 (15.9%) and (b) $\overline{X}$ - S at an exceedence probability of 0.841 (84.1%).

3. Draw a straight line through these two points; the accuracy of the graphing can be checked by ensuring that the line passes through the point defined by $\overline{X}$ at an exceedence probability of 0.50 (50%).

The straight line represents the assumed normal population. It can be used either to make probability estimates for given values of X or to estimate values of X for given exceedence probabilities.

4.3.1.3 Plotting Sample Data

Before a computed frequency curve is used to make estimates of either flood magnitudes or exceedence probabilities, the assumed population should be verified by plotting the data. The following steps are used to plot the data:

1. Rank the flood series in descending order, with the largest flood having a rank of 1 and the smallest flood having a rank of n.

2. Use the rank (i) with a plotting position formula such as Equation 4.21, and compute the plotting probabilities for each flood.

3. Plot the magnitude X against the corresponding plotting probability.

If the data follow the trend of the assumed population line, one usually assumes that the data are normally distributed. It is not uncommon for the sample points on the upper and lower ends to deviate from the straight line. Deciding whether or not to accept the computed straight line as the population is based on experience rather than an objective criterion.

4.3.1.4 Estimation with the Frequency Curve

Once the population line has been verified and accepted, the line can be used for estimation. While graphical estimates are acceptable for some work, it is often important to use Equations 4.24 and 4.25 in estimating flood magnitudes or probabilities. To make a probability estimate p for a given magnitude, use the following procedure:

1. Use Equation 4.24 to compute the value of the standard normal deviate.

2. Enter Table 4.9 with the value of z and obtain the exceedence probability.

To make estimates of the magnitude for a given exceedence probability, use the following procedure:

1. Enter Table 4.9 with the exceedence probability and obtain the corresponding value of z.

2. Use Equation 4.25 with $\overline{X}$, S, and z to compute the magnitude X.

Table 4.9. Probabilities of the Cumulative Standard Normal Distribution for Selected Values of the Standard Normal Deviate (z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table 4.9. Probabilities of the Cumulative Standard Normal Distribution for Selected Values of the Standard Normal Deviate (z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Example 4.6. To illustrate the use of these concepts, consider the data of Table 4.10. These data are the annual peak floods for the Medina River near San Antonio, Texas, for the period 1940-1982 (43 years of record) ranked from largest to smallest. Using Equations 4.12 and 4.13 for mean and standard deviation, respectively, and assuming the data are normally distributed, the 10-year and 100-year floods are computed as follows using SI and CU units:

Variable	Value in SI	Value in CU
$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$	$\frac{8,040}{43} = 187.0\ m^3/s$	$\frac{283,900}{43} = 6,602\ ft^3/s$
$S = \overline{X}\left[\frac{\sum_{i=1}^{n}\left(\frac{X_i}{\overline{X}} - 1\right)^2}{n - 1}\right]^{0.5}$	$187.0\left[\frac{48.22}{43-1}\right]^{0.5} = 200.4\ m^3/s$	$6,602\left[\frac{48.22}{43-1}\right]^{0.5} = 7,074\ ft^3/s$
$X_{10} = \overline{X} + z_{10}S$	$187.0 + 1.282\ (200.4)$ $= 444\ m^3/s$	$6,602 + 1.282\ (7,074)$ $= 15,700\ ft^3/s$
$X_{100} = \overline{X} + z_{100}S$	$187.0 + 2.326\ (200.4)$ $= 653\ m^3/s$	$6,602 + 2.326(7,074)$ $= 23,100\ ft^3/s$

When plotted on arithmetic probability scales, these two points are sufficient to establish the straight line on Figure 4.12 represented by Equation 4.26. For comparison, the measured discharges are plotted in Figure 4.12 using the Weibull plotting-position formula. The correspondence between the normal frequency curve and the actual data is poor. Obviously, the data are not normally distributed. Using Equations 4.14 and 4.15 to estimate the variance and skew, it becomes clear that the data have a large skew while the normal distribution has a skew of zero. This explains the poor correspondence in this case.

Variable	Value in SI	Value in CU
$V = \frac{S}{\overline{X}}$	$\frac{200.4}{187.0} = 1.072$	$\frac{7,074}{6,602} = 1.072$
$G = \frac{n\sum\left(\frac{X_i}{\overline{X}} - 1\right)^3}{(n-1)(n-2)V^3}$	$\frac{43(117.4)}{42(41)(1.072)^3} = 2.38$	$\frac{43(117.4)}{42(41)(1.072)^3} = 2.38$

Table 4.10. Frequency Analysis Computations for the Normal Distribution: Medina River, TX (Gage 08181500)

Year	Rank	Plotting Probability	Annual Maximum (m^3/s)	Annual Maximum (ft^3/s)	$X/\bar{X}$	$(X/\bar{X})-1$	$[(X/\bar{X})-1]^2$	$[(X-\bar{X})-1]^3$
1973	1	0.023	903.4	31,900	4.832	3.832	14.681	56.250
1946	2	0.045	900.6	31,800	4.816	3.816	14.565	55.586
1942	3	0.068	495.6	17,500	2.651	1.651	2.724	4.496
1949	4	0.091	492.8	17,400	2.635	1.635	2.674	4.374
1981	5	0.114	410.6	14,500	2.196	1.196	1.431	1.711
1968	6	0.136	371.0	13,100	1.984	0.984	0.968	0.953
1943	7	0.159	342.7	12,100	1.833	0.833	0.693	0.577
1974	8	0.182	274.1	9,680	1.466	0.466	0.217	0.101
1978	9	0.205	267.3	9,440	1.430	0.430	0.185	0.079
1958	10	0.227	261.1	9,220	1.396	0.396	0.157	0.062
1982	11	0.250	231.1	8,160	1.236	0.236	0.056	0.013
1976	12	0.273	212.7	7,510	1.137	0.137	0.019	0.003
1941	13	0.295	195.1	6,890	1.044	0.044	0.002	0.000
1972	14	0.318	180.1	6,360	0.963	-0.037	0.001	0.000
1950	15	0.341	160.3	5,660	0.857	-0.143	0.020	-0.003
1967	16	0.364	155.2	5,480	0.830	-0.170	0.029	-0.005
1965	17	0.386	153.8	5,430	0.822	-0.178	0.032	-0.006
1957	18	0.409	146.7	5,180	0.785	-0.215	0.046	-0.010
1953	19	0.432	140.5	4,960	0.751	-0.249	0.062	-0.015
1979	20	0.455	134.5	4,750	0.719	-0.281	0.079	-0.022
1977	21	0.477	130.8	4,620	0.700	-0.300	0.090	-0.027
1975	22	0.500	117.0	4,130	0.626	-0.374	0.140	-0.053
1962	23	0.523	112.1	3,960	0.600	-0.400	0.160	-0.064
1945	24	0.545	100.3	3,540	0.536	-0.464	0.215	-0.100
1970	25	0.568	95.2	3,360	0.509	-0.491	0.241	-0.118
1959	26	0.591	94.9	3,350	0.507	-0.493	0.243	-0.120
1960	27	0.614	90.6	3,200	0.485	-0.515	0.266	-0.137
1961	28	0.636	86.4	3,050	0.462	-0.538	0.289	-0.156
1971	29	0.659	83.5	2,950	0.447	-0.553	0.306	-0.169
1969	30	0.682	77.3	2,730	0.413	-0.587	0.344	-0.202
1940	31	0.705	71.9	2,540	0.385	-0.615	0.379	-0.233
1966	32	0.727	61.2	2,160	0.327	-0.673	0.453	-0.305
1951	33	0.750	60.9	2,150	0.326	-0.674	0.455	-0.307
1964	34	0.773	60.6	2,140	0.324	-0.676	0.457	-0.309
1948	35	0.795	58.1	2,050	0.310	-0.690	0.475	-0.328
1944	36	0.818	56.6	2,000	0.303	-0.697	0.486	-0.339
1980	37	0.841	56.1	1,980	0.300	-0.700	0.490	-0.343
1956	38	0.864	49.6	1,750	0.265	-0.735	0.540	-0.397
1947	39	0.886	41.6	1,470	0.223	-0.777	0.604	-0.470
1955	40	0.909	34.0	1,200	0.182	-0.818	0.670	-0.548
1963	41	0.932	25.2	890	0.135	-0.865	0.749	-0.648
1954	42	0.955	24.5	865	0.131	-0.869	0.755	-0.656
1952	43	0.977	22.7	801	0.121	-0.879	0.772	-0.679
Total			**8,040.3**	**283,906**			**48.22**	**117.4**

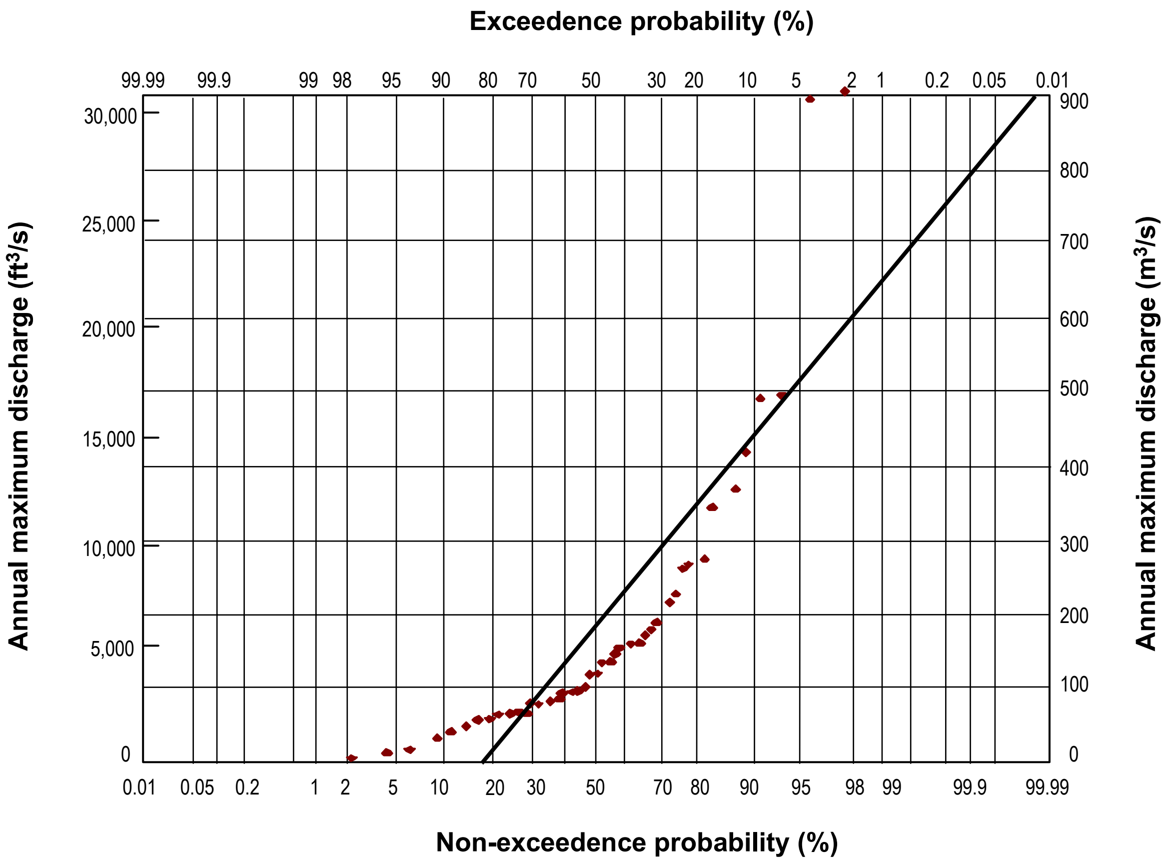

Figure 4.12. Normal distribution frequency curve, Medina River

4.3.2 Log-Normal Distribution

The log-normal distribution has the same characteristics as the normal distribution except that the dependent variable, X, is replaced with its logarithm. The characteristics of the log-normal distribution are that it is bounded on the left by zero and it has a pronounced positive skew. These are both characteristics of many of the frequency distributions that result from an analysis of hydrologic data.

If a logarithmic transformation is performed on the normal distribution function, the resulting logarithmic distribution is normally distributed. This enables the z values tabulated in Tables 4-8 and 4-9 for a standard normal distribution to be used in a log-normal frequency analysis (Table 4.10). A three-parameter log-normal distribution exists, which makes use of a shift parameter. Only the zero-skew log-normal distribution will be discussed. As was the case with the normal distribution, log-normal probability scales have been developed, where the plot of the cumulative distribution function is a straight line. This scale uses a transformed horizontal scale based upon the probability function of the normal distribution and a logarithmic vertical scale. If the logarithms of the peak flows are normally distributed, the data will plot as a straight line according to the equation:

$$Y = \log X = \overline{Y} + K\, S_y \qquad (4.27)$$

where,

$\overline{Y}$ = average of the logarithms of X

S_y = standard deviation of the logarithms.

4.3.2.1 Procedure

The procedure for developing the graph of the log-normal distribution is similar to that for the normal distribution:

1. Transform the values of the flood series X by taking logarithms: Y = log X.

2. Compute the log mean ($\overline{Y}$) and log standard deviation (S_y) using the logarithms.

3. Using $\overline{Y}$ and S_y, compute $10^{\overline{Y} + S_y}$ and $10^{\overline{Y} - S_y}$. Using logarithmic frequency paper, plot these two values at exceedence probabilities of 0.159 (15.9%) and 0.841 (84.1%), respectively.

4. Draw a straight line through the two points.

The data points can now be plotted on the logarithmic probability paper using the same procedure as outlined for the normal distribution. Specifically, the flood magnitudes are plotted against the probabilities from a plotting position formula (e.g., Equation 4.21).

4.3.2.2 Estimation

Graphical estimates of either flood magnitudes or probabilities can be taken directly from the line representing the assumed log-normal distribution. Values can also be computed using either:

$$z = \frac{Y - \overline{Y}}{S_y} \qquad (4.28)$$

to obtain a probability for the logarithm of a given magnitude (Y = log X) or:

$$Y = \overline{Y} + z\, S_y \qquad (4.29)$$

to obtain a magnitude for a given probability. The value computed with Equation 4.29 must be transformed:

$$X = 10^Y \qquad (4.30)$$

Two useful relations are also available to approximate the mean and the standard deviation of the logarithms, $\overline{Y}$ and S_y, from $\overline{X}$ and S of the original variables. These equations are

$$\overline{Y} = 0.5 \log\left(\frac{\overline{X}^4}{\overline{X}^2 + S^2}\right) \qquad (4.31)$$

and

$$S_y = \left[log \left(\frac{S^2 + \overline{X}^2}{\overline{X}^2} \right) \right]^{0.5} \tag{4.32}$$

<u>Example 4.7</u>. The log-normal distribution will be illustrated using the 43-year record from the Medina River shown in Table 4.11. Mean and standard deviation are calculated as follows:

Variable	Value in SI	Value in CU
$\overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$	$= \frac{89.92}{43} = 2.091$	$= \frac{156.48}{43} = 3.639$
$S_y = \overline{Y} \left[\frac{\sum_{i=1}^{n} \left(\frac{Y_i}{\overline{Y}} - 1 \right)^2}{n - 1} \right]^{0.5}$	$= 2.091 \left(\frac{1.492}{42} \right)^{0.5} = 0.394$	$= 3.639 \left(\frac{0.493}{42} \right)^{0.5} = 0.394$

Assuming the distribution of the logs is normal, the 10-year and 100-year floods are:

Variable	Value in SI	Value in CU
$Y_{10} = \overline{Y} + z_{10} S_y$	$=2.091+1.282\ (0.394)=2.596$	$=3.639+1.282\ (0.394)=4.144$
$X_{10} = 10^{Y_{10}}$	$= 10^{2.596} = 394\ m^3/s$	$= 10^{4.144} = 13{,}900\ ft^3/s$
$Y_{100} = \overline{Y} + z_{100} S_y$	$=2.091+2.326(0.394)=3.007$	$=3.639+2.326(0.394)=4.555$
$X_{100} = 10^{Y_{100}}$	$= 10^{3.007} = 1{,}020\ m^3/s$	$= 10^{4.555} = 35{,}900\ ft^3/s$

The measured flood data are also plotted on log-probability scales in Figure 4.13 together with the fitted log-normal distribution. (Note: When plotting X on the log scale, the actual values of X are plotted rather than their logarithms since the log-scale effectively transforms the data to their respective logarithms.) Figure 4.13 shows that the log-normal distribution fits the actual data better than the normal distribution shown in Figure 4.12. A smaller skew, as calculated below, explains the improved fit:

Variable	Value in SI	Value in CU
$V_y = \frac{S_y}{\overline{Y}}$	$= \frac{0.394}{2.091} = 0.188$	$= \frac{0.394}{3.639} = 0.108$
$G_y = \frac{n \sum_{i=1}^{n} \left(\frac{Y_i}{\overline{Y}} - 1 \right)^3}{(n-1)(n-2) V_y^3}$	$= \frac{43(0.06321)}{(42)(41)(0.188)^3} = 0.24$	$= \frac{43(0.01199)}{(42)(41)(0.108)^3} = 0.24$

Table 4.11. Frequency Analysis Computations for the Log-Normal Distribution: Medina River

(a) SI Units

Year	Rank	Plotting Probability	Annual Max.(X) (m^3/s)	Y = log(X)	$Y/\bar{Y}$	$[(Y/\bar{Y})-1]$	$[(Y/\bar{Y})-1]^2$	$[(Y/\bar{Y})-1]^3$
1973	1	0.023	903.4	2.956	1.413	0.413	0.1709	0.0707
1946	2	0.045	900.6	2.955	1.413	0.413	0.1704	0.0703
1942	3	0.068	495.6	2.695	1.289	0.289	0.0834	0.0241
1949	4	0.091	492.8	2.693	1.288	0.288	0.0827	0.0238
1981	5	0.114	410.6	2.613	1.250	0.250	0.0624	0.0156
1968	6	0.136	371.0	2.569	1.229	0.229	0.0523	0.0120
1943	7	0.159	342.7	2.535	1.212	0.212	0.0450	0.0095
1974	8	0.182	274.1	2.438	1.166	0.166	0.0275	0.0046
1978	9	0.205	267.3	2.427	1.161	0.161	0.0258	0.0041
1958	10	0.227	261.1	2.417	1.156	0.156	0.0242	0.0038
1982	11	0.250	231.1	2.364	1.130	0.130	0.0170	0.0022
1976	12	0.273	212.7	2.328	1.113	0.113	0.0128	0.0014
1941	13	0.295	195.1	2.290	1.095	0.095	0.0091	0.0009
1972	14	0.318	180.1	2.256	1.079	0.079	0.0062	0.0005
1950	15	0.341	160.3	2.205	1.054	0.054	0.0030	0.0002
1967	16	0.364	155.2	2.191	1.048	0.048	0.0023	0.0001
1965	17	0.386	153.8	2.187	1.046	0.046	0.0021	0.0001
1957	18	0.409	146.7	2.166	1.036	0.036	0.0013	0.0000
1953	19	0.432	140.5	2.148	1.027	0.027	0.0007	0.0000
1979	20	0.455	134.5	2.129	1.018	0.018	0.0003	0.0000
1977	21	0.477	130.8	2.117	1.012	0.012	0.0001	0.0000
1975	22	0.500	117.0	2.068	0.989	-0.011	0.0001	0.0000
1962	23	0.523	112.1	2.050	0.980	-0.020	0.0004	0.0000
1945	24	0.545	100.3	2.001	0.957	-0.043	0.0019	-0.0001
1970	25	0.568	95.2	1.978	0.946	-0.054	0.0029	-0.0002
1959	26	0.591	94.9	1.977	0.945	-0.055	0.0030	-0.0002
1960	27	0.614	90.6	1.957	0.936	-0.064	0.0041	-0.0003
1961	28	0.636	86.4	1.936	0.926	-0.074	0.0055	-0.0004
1971	29	0.659	83.5	1.922	0.919	-0.081	0.0066	-0.0005
1969	30	0.682	77.3	1.888	0.903	-0.097	0.0094	-0.0009
1940	31	0.705	71.9	1.857	0.888	-0.112	0.0126	-0.0014
1966	32	0.727	61.2	1.787	0.854	-0.146	0.0212	-0.0031
1951	33	0.750	60.9	1.785	0.853	-0.147	0.0215	-0.0032
1964	34	0.773	60.6	1.783	0.852	-0.148	0.0218	-0.0032
1948	35	0.795	58.1	1.764	0.843	-0.157	0.0245	-0.0038
1944	36	0.818	56.6	1.753	0.838	-0.162	0.0261	-0.0042
1980	37	0.841	56.1	1.749	0.836	-0.164	0.0268	-0.0044
1956	38	0.864	49.6	1.695	0.811	-0.189	0.0359	-0.0068
1947	39	0.886	41.6	1.619	0.774	-0.226	0.0509	-0.0115
1955	40	0.909	34.0	1.531	0.732	-0.268	0.0717	-0.0192
1963	41	0.932	25.2	1.401	0.670	-0.330	0.1088	-0.0359
1954	42	0.955	24.5	1.389	0.664	-0.336	0.1127	-0.0378
1952	43	0.977	22.7	1.355	0.648	-0.352	0.1239	-0.0436
Total			**8,040.3**	**89.92**			**1.992**	**0.06321**

Table 4.11. Frequency Analysis Computations for the Log-Normal Distribution: Medina River (Continued)

(b) CU Units

Year	Rank	Plotting Probability	Annual Max.(x) (ft^3/s)	Y = Log(X)	$Y/\bar{Y}$	$[(Y/\bar{Y})-1]$	$[(Y/\bar{Y})-1]^2$	$[(Y/\bar{Y})-1]^3$
1973	1	0.023	31,900	4.504	1.238	0.238	0.0565	0.0134
1946	2	0.045	31,800	4.502	1.237	0.237	0.0563	0.0133
1942	3	0.068	17,500	4.243	1.166	0.166	0.0275	0.0046
1949	4	0.091	17,400	4.241	1.165	0.165	0.0273	0.0045
1981	5	0.114	14,500	4.161	1.144	0.144	0.0206	0.0030
1968	6	0.136	13,100	4.117	1.131	0.131	0.0173	0.0023
1943	7	0.159	12,100	4.083	1.122	0.122	0.0149	0.0018
1974	8	0.182	9,680	3.986	1.095	0.095	0.0091	0.0009
1978	9	0.205	9,440	3.975	1.092	0.092	0.0085	0.0008
1958	10	0.227	9,220	3.965	1.089	0.089	0.0080	0.0007
1982	11	0.250	8,160	3.912	1.075	0.075	0.0056	0.0004
1976	12	0.273	7,510	3.876	1.065	0.065	0.0042	0.0003
1941	13	0.295	6,890	3.838	1.055	0.055	0.0030	0.0002
1972	14	0.318	6,360	3.803	1.045	0.045	0.0020	0.0001
1950	15	0.341	5,660	3.753	1.031	0.031	0.0010	0.0000
1967	16	0.364	5,480	3.739	1.027	0.027	0.0007	0.0000
1965	17	0.386	5,430	3.735	1.026	0.026	0.0007	0.0000
1957	18	0.409	5,180	3.714	1.021	0.021	0.0004	0.0000
1953	19	0.432	4,960	3.695	1.015	0.015	0.0002	0.0000
1979	20	0.455	4,750	3.677	1.010	0.010	0.0001	0.0000
1977	21	0.477	4,620	3.665	1.007	0.007	0.0000	0.0000
1975	22	0.500	4,130	3.616	0.994	-0.006	0.0000	0.0000
1962	23	0.523	3,960	3.598	0.989	-0.011	0.0001	0.0000
1945	24	0.545	3,540	3.549	0.975	-0.025	0.0006	0.0000
1970	25	0.568	3,360	3.526	0.969	-0.031	0.0010	0.0000
1959	26	0.591	3,350	3.525	0.969	-0.031	0.0010	0.0000
1960	27	0.614	3,200	3.505	0.963	-0.037	0.0014	0.0000
1961	28	0.636	3,050	3.484	0.957	-0.043	0.0018	-0.0001
1971	29	0.659	2,950	3.470	0.953	-0.047	0.0022	-0.0001
1969	30	0.682	2,730	3.436	0.944	-0.056	0.0031	-0.0002
1940	31	0.705	2,540	3.405	0.936	-0.064	0.0041	-0.0003
1966	32	0.727	2,160	3.334	0.916	-0.084	0.0070	-0.0006
1951	33	0.750	2,150	3.332	0.916	-0.084	0.0071	-0.0006
1964	34	0.773	2,140	3.330	0.915	-0.085	0.0072	-0.0006
1948	35	0.795	2,050	3.312	0.910	-0.090	0.0081	-0.0007
1944	36	0.818	2,000	3.301	0.907	-0.093	0.0086	-0.0008
1980	37	0.841	1,980	3.297	0.906	-0.094	0.0089	-0.0008
1956	38	0.864	1,750	3.243	0.891	-0.109	0.0118	-0.0013
1947	39	0.886	1,470	3.167	0.870	-0.130	0.0168	-0.0022
1955	40	0.909	1,200	3.079	0.846	-0.154	0.0237	-0.0036
1963	41	0.932	890	2.949	0.810	-0.190	0.0359	-0.0068
1954	42	0.955	865	2.937	0.807	-0.193	0.0372	-0.0072
1952	43	0.977	801	2.903	0.798	-0.202	0.0409	-0.0083
Total			**283,906**	**156.48**			**0.492**	**0.0121**

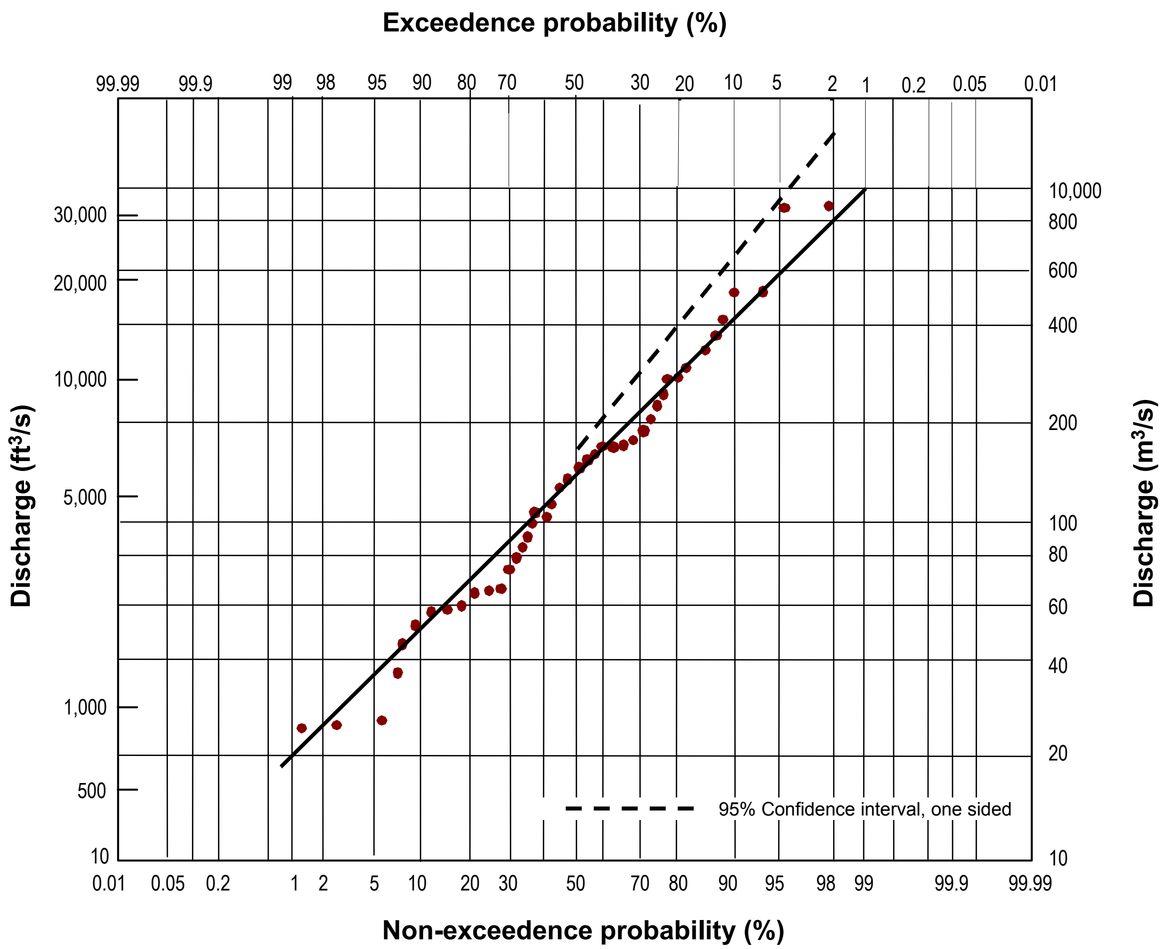

Figure 4.13. Log-normal distribution frequency curve (solid line) and one-sided upper confidence interval (dashed line)

4.3.3 Gumbel Extreme Value Distribution

The Gumbel extreme value distribution, sometimes called the double-exponential distribution of extreme values, can also be used to describe the distribution of hydrologic variables, especially peak discharges. It is based upon the assumption that the cumulative frequency distribution of the largest values of samples drawn from a large population can be described by the following equation:

$$F(X) = e^{-e^{\alpha(X-\beta)}} \tag{4.33}$$

where,

$$\alpha = \frac{1.281}{S} \quad (4.33a)$$

$$\beta = \overline{X} - 0.450\ S \quad (4.33b)$$

In a manner analogous to that of the normal distribution, values of the distribution function can be computed from Equation 4.33. Frequency factor values K are tabulated for convenience in Table 4.12 for use in Equation 4.26.

Table 4.12. Frequency Factors (K) for the Gumbel Extreme Value Distribution

Sample Size n	Exceedence Probability in %						
	50.0	20.0	10.0	4.0	2.0	1.0	0.2
	Corresponding Return Period in Years						
	2	5	10	25	50	100	500
10	-0.1355	1.0581	1.8483	2.8468	3.5876	4.3228	6.0219
15	-0.1433	0.9672	1.7025	2.6315	3.3207	4.0048	5.5857
20	-0.1478	0.9186	1.6247	2.5169	3.1787	3.8357	5.3538
25	-0.1506	0.8879	1.5755	2.4442	3.0887	3.7285	5.2068
30	-0.1525	0.8664	1.5410	2.3933	3.0257	3.6533	5.1038
35	-0.1540	0.8504	1.5153	2.3555	2.9789	3.5976	5.0273
40	-0.1552	0.8379	1.4955	2.3262	2.9426	3.5543	4.9680
45	-0.1561	0.8280	1.4795	2.3027	2.9134	3.5196	4.9204
50	-0.1568	0.8197	1.4662	2.2831	2.8892	3.4907	4.8808
55	-0.1574	0.8128	1.4552	2.2668	2.8690	3.4667	4.8478
60	-0.1580	0.8069	1.4457	2.2529	2.8517	3.4460	4.8195
65	-0.1584	0.8019	1.4377	2.2410	2.8369	3.4285	4.7955
70	-0.1588	0.7973	1.4304	2.2302	2.8236	3.4126	4.7738
75	-0.1592	0.7934	1.4242	2.2211	2.8123	3.3991	4.7552
80	-0.1595	0.7899	1.4186	2.2128	2.8020	3.3869	4.7384
85	-0.1598	0.7868	1.4135	2.2054	2.7928	3.3759	4.7234
90	-0.1600	0.7840	1.4090	2.1987	2.7845	3.3660	4.7098
95	-0.1602	0.7815	1.4049	2.1926	2.7770	3.3570	4.6974
100	-0.1604	0.7791	1.4011	2.1869	2.7699	3.3487	4.6860

Characteristics of the Gumbel extreme-value distribution are that the mean flow, $\overline{X}$, occurs at the return period of T_r = 2.33 years and that it has a positive skew (i.e., it is skewed toward the high flows or extreme values).

As was the case with the two previous distributions, special probability scales have been developed so that sample data, if they are distributed according to Equation 4.33, will plot as a straight line. Most USGS offices have prepared forms with these axis on which the horizontal scale has been transformed by the double-logarithmic transform of Equation 4.33.

<u>Example 4.8</u>. Peak flow data for the Medina River can be fit with a Gumbel distribution using Equation 4.26 and values of K from Table 4.12. The mean and standard deviation were calculated earlier as 187.0 m^3/s (6,602 ft^3/s) and 200.4 m^3/s (7,074 ft^3/s), respectively. The 10-year flood computed from the Gumbel distribution is:

Variable	Value in SI	Value in CU
$X_{10} = \overline{X} + KS$	*187.0 + 1.486 (200.4) = 485 m^3/s*	*6,602 + 1.486 (7,074) = 17,100 ft^3/s*

and the 100-year flood is:

Variable	Value in SI	Value in CU
$X_{100} = \overline{X} + KS$	*187.0 + 3.534 (200.4) = 895 m^3/s*	*6,602 + 3.534 (7,074) = 31,600 ft^3/s*

Plotted on the Gumbel graph in Figure 4.14 are the actual flood data and the computed frequency curve.

Although the Gumbel distribution is skewed positively, it does not account directly for the computed skew of the data, but does predict the high flows reasonably well. However, the entire curve fit is not much better than that obtained with the normal distribution, indicating this peak flow series is not distributed according to the double-exponential distribution of Equation 4.33.

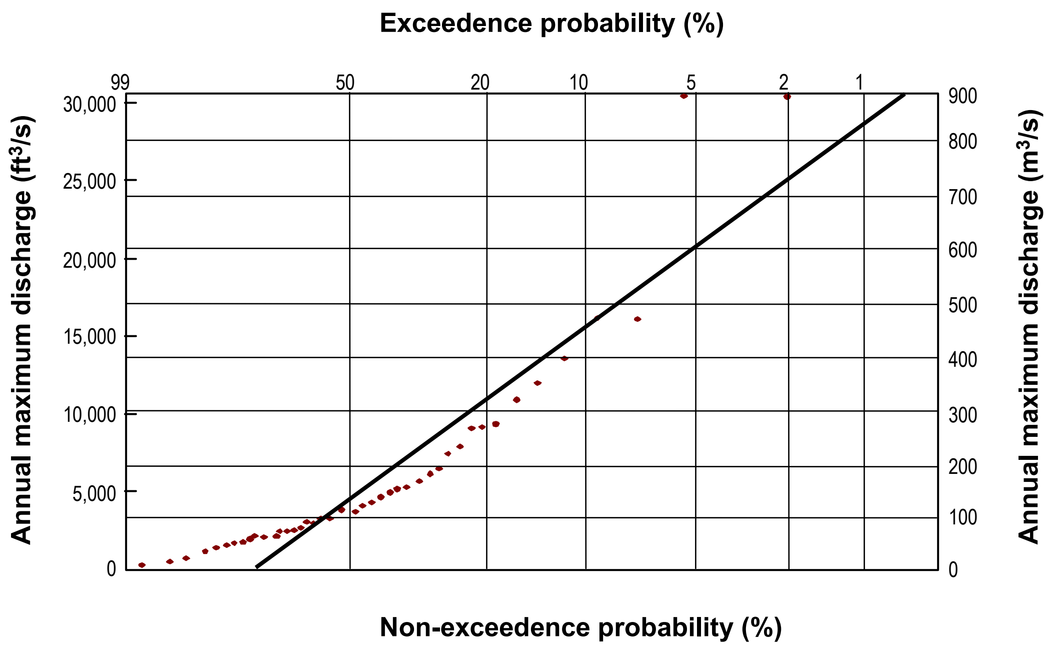

Figure 4.14. Gumbel extreme value distribution frequency curve, Medina River

4.3.4 Log-Pearson Type III Distribution

Another distribution that has found wide application in hydrologic analysis is the log-Pearson Type III distribution. The log-Pearson Type III distribution is a three-parameter gamma distribution with a logarithmic transform of the variable. It is widely used for flood analyses because the data quite frequently fit the assumed population. It is this flexibility that led the Interagency Advisory Committee on Water Data to recommend its use as the standard distribution for flood frequency studies by all U.S. Government agencies. Thomas (1985) describes the motivation for adopting the log-Pearson Type III distribution and the events leading up to USGS Bulletin 17B (1982).

The log-Pearson Type III distribution differs from most of the distributions discussed above in that three parameters (mean, standard deviation, and coefficient of skew) are necessary to describe the distribution. By judicious selection of these three parameters, it is possible to fit just about any shape of distribution. An extensive treatment on the use of this distribution in the determination of flood frequency distributions is presented in USGS Bulletin 17B, "Guidelines for Determining Flood Frequency" by the Interagency Advisory Committee on Water Data, revised March 1982. The Bulletin 17B procedure assumes the logarithms of the annual peak flows are Pearson Type III distributed rather than assuming the untransformed data are log-Pearson Type III. Kite (1988) has a good description of the two approaches.

An abbreviated table of the log-Pearson Type III distribution function is given in Table 4.13. (Extensive tables that reduce the amount of interpolation can be found in USGS Bulletin 17B, 1982.) Using the mean, standard deviation, and skew coefficient for any set of log-transformed annual peak flow data, in conjunction with Table 4.13, the flood with any exceedence frequency can be computed from the equation:

$$\hat{Y} = log\ X = \overline{Y} + KS_y \tag{4.34}$$

where $\hat{Y}$ is the predicted value of log X, $\overline{Y}$ and S_y are as previously defined, and K is a function of the exceedence probability and the coefficient of skew.

Again, it would be possible to develop special probability scales, so that the log-Pearson Type III distribution would plot as a straight line. However, the log-Pearson Type III distribution can assume a variety of shapes so that a separate probability scale would be required for each different shape. Since this is impractical, log-Pearson Type III distributions are usually plotted on log-normal probability scales even though the plotted frequency distribution may not be a straight line. It is a straight line only when the skew of the logarithms is zero.

4.3.4.1 Procedure

The procedure for fitting the log-Pearson Type III distribution is similar to that for the normal and log-normal. The specific steps for making a basic log-Pearson Type III analysis without any of the optional adjustments are as follows:

1. Make a logarithmic transform of all flows in the series (Y_i = log X_i).

2. Compute the mean ($\overline{Y}$), standard deviation (S_y), and standardized skew (G) of the logarithms using Equations 4.12, 4.13, and 4.15, respectively. Round the skew to the nearest tenth (e.g., 0.32 is rounded to 0.3).

3. Since the log-Pearson Type III curve with a nonzero skew does not plot as a straight line, it is necessary to use more than two points to draw the curve. The curvature of the line will increase as the absolute value of the skew increases, so more points will be needed for larger skew magnitudes.

4. Compute the logarithmic value $\hat{Y}$ for each exceedence frequency using Equation 4.34.

5. Transform the computed values of step 4 to discharges using equation 4.35:

$$\hat{X} = 10^{\hat{Y}} \tag{4.35}$$

   in which $\hat{X}$ is the computed discharge for the assumed log-Pearson Type III population.

6. Plot the points of step 5 on logarithmic probability paper and draw a smooth curve through the points.

The sample data can be plotted on the paper using a plotting position formula to obtain the exceedence probability. The computed curve can then be verified, and, if acceptable, it can be used to make estimates of either a flood probability or flood magnitude.

4.3.4.2 Estimation

In addition to graphical estimation, estimates can be made with the mathematical model of Equation 4.34. To compute a magnitude for a given probability, the procedure is the same as that in steps 3 to 5 above. To estimate the probability for a given magnitude X, the value is transformed using the logarithm (Y = log X) and then Equation 4.34 is algebraically transformed to compute K:

$$K = \frac{Y - \overline{Y}}{S_y} \tag{4.36}$$

The computed value of K should be compared to the K values of Table 4.13 for the standardized skew and a value of the probability interpolated from the probability values on Table 4.13; linear interpolation is acceptable.

Example 4.9. The log-Pearson Type III distribution will be illustrated using the Medina River flood data (Table 4.11). Three cases will be computed: station skew, generalized skew, and weighted skew. Table 4.13 and Equation 4.34 are used to compute values of the log-Pearson Type III distribution for the 10- and 100-year flood using the parameters, $\overline{Y}$, S_y, and G for the Medina River flood data. (To help define the distribution, the 2-, 5-, 25-, and 50-year floods have also been computed in Table 4.14.) Rounding the station skew of 0.236 to 0.2, the log-Pearson Type III distribution estimates of the 100- and 10-year floods are 1,160 m^3/s (41,000 ft^3/s) and 402 m^3/s (14,200 ft^3/s), respectively. The log-Pearson Type III distribution (G = 0.2) and the actual data from Table 4.11 are plotted in Figure 4.15 on log-normal probability scales.

The generalized skew coefficient for the Medina River is -0.252, which can be rounded to -0.3. Using this option, the 10- and 100-year floods for the Medina River are estimated as shown in Table 4.15. This log-Pearson Type III distribution (generalized skew coefficient, $\overline{G}$ = -0.3) is also plotted on Figure 4.15.

To illustrate the use of weighted skew, the station and generalized skews have already been determined to be G = 0.236 and $\overline{G}$ = -0.252, respectively. The mean-square error of $\overline{G}$, $MSE_{\overline{G}}$, is 0.302 and from Equation 4.17, MSE_G = 0.136. From Equation 4.16, the weighted skew is:

$$G_w = \frac{0.302(0.236) + 0.136(-0.252)}{0.302 + 0.136} = 0.084$$

which is rounded to 0.1 when obtaining values from Table 4.13. Values for selected return periods are given in Table 4.16.

Table 4.13. Frequency Factors (K) for the Log-Pearson Type III Distribution

	Skew						
Prob.	**-2.0**	**-1.9**	**-1.8**	**-1.7**	**-1.6**	**-1.5**	**-1.4**
0.9999	-8.21034	-7.98888	-7.76632	-7.54272	-7.31818	-7.09277	-6.86661
0.9995	-6.60090	-6.44251	-6.28285	-6.12196	-5.95990	-5.79673	-5.63252
0.9990	-5.90776	-5.77549	-5.64190	-5.50701	-5.37087	-5.23353	-5.09505
0.9980	-5.21461	-5.10768	-4.99937	-4.88971	-4.77875	-4.66651	-4.55304
0.9950	-4.29832	-4.22336	-4.14700	-4.06926	-3.99016	-3.90973	-3.82798
0.9900	-3.60517	-3.55295	-3.49935	-3.44438	-3.38804	-3.33035	-3.27134
0.9800	-2.91202	-2.88091	-2.84848	-2.81472	-2.77964	-2.74325	-2.70556
0.9750	-2.68888	-2.66413	-2.63810	-2.61076	-2.58214	-2.55222	-2.52102
0.9600	-2.21888	-2.20670	-2.19332	-2.17873	-2.16293	-2.14591	-2.12768
0.9500	-1.99573	-1.98906	-1.98124	-1.97227	-1.96213	-1.95083	-1.93836
0.9000	-1.30259	-1.31054	-1.31760	-1.32376	-1.32900	-1.33330	-1.33665
0.8000	-0.60944	-0.62662	-0.64335	-0.65959	-0.67532	-0.69050	-0.70512
0.7000	-0.20397	-0.22250	-0.24094	-0.25925	-0.27740	-0.29535	-0.31307
0.6000	0.08371	0.06718	0.05040	0.03344	0.01631	-0.00092	-0.01824
0.5704	0.15516	0.13964	0.12381	0.10769	0.09132	0.07476	0.05803
0.5000	0.30685	0.29443	0.28150	0.26808	0.25422	0.23996	0.22535
0.4296	0.43854	0.43008	0.42095	0.41116	0.40075	0.38977	0.37824
0.4000	0.48917	0.48265	0.47538	0.46739	0.45873	0.44942	0.43949
0.3000	0.64333	0.64453	0.64488	0.64436	0.64300	0.64080	0.63779
0.2000	0.77686	0.78816	0.79868	0.80837	0.81720	0.82516	0.83223
0.1000	0.89464	0.91988	0.94496	0.96977	0.99418	1.01810	1.04144
0.0500	0.94871	0.98381	1.01973	1.05631	1.09338	1.13075	1.16827
0.0400	0.95918	0.99672	1.03543	1.07513	1.11566	1.15682	1.19842
0.0250	0.97468	1.01640	1.06001	1.10537	1.15229	1.20059	1.25004
0.0200	0.97980	1.02311	1.06864	1.11628	1.16584	1.21716	1.26999
0.0100	0.98995	1.03695	1.08711	1.14042	1.19680	1.25611	1.31815
0.0050	0.99499	1.04427	1.09749	1.15477	1.21618	1.28167	1.35114
0.0020	0.99800	1.04898	1.10465	1.16534	1.23132	1.30279	1.37981
0.0010	0.99900	1.05068	1.10743	1.16974	1.23805	1.31275	1.39408
0.0005	0.99950	1.05159	1.10901	1.17240	1.24235	1.31944	1.40413
0.0001	0.99990	1.05239	1.11054	1.17520	1.24728	1.32774	1.41753

Table 4.13. Frequency Factors (K) for the Log-Pearson Type III Distribution (Cont'd)

Prob.	Skew						
	-1.3	-1.2	-1.1	-1.0	-0.9	-0.8	-0.7
0.9999	-6.63980	-6.41249	-6.18480	-5.95691	-5.72899	-5.50124	-5.27389
0.9995	-5.46735	-5.30130	-5.13449	-4.96701	-4.79899	-4.63057	-4.46189
0.9990	-4.95549	-4.81492	-4.67344	-4.53112	-4.38807	-4.24439	-4.10022
0.9980	-4.43839	-4.32263	-4.20582	-4.08802	-3.96932	-3.84981	-3.72957
0.9950	-3.74497	-3.66073	-3.57530	-3.48874	-3.40109	-3.31243	-3.22281
0.9900	-3.21103	-3.14944	-3.08660	-3.02256	-2.95735	-2.89101	-2.82359
0.9800	-2.66657	-2.62631	-2.58480	-2.54206	-2.49811	-2.45298	-2.40670
0.9750	-2.48855	-2.45482	-2.41984	-2.38364	-2.34623	-2.30764	-2.26790
0.9600	-2.10823	-2.08758	-2.06573	-2.04269	-2.01848	-1.99311	-1.96660
0.9500	-1.92472	-1.90992	-1.89395	-1.87683	-1.85856	-1.83916	-1.81864
0.9000	-1.33904	-1.34047	-1.34092	-1.34039	-1.33889	-1.33640	-1.33294
0.8000	-0.71915	-0.73257	-0.74537	-0.75752	-0.76902	-0.77986	-0.79002
0.7000	-0.33054	-0.34772	-0.36458	-0.38111	-0.39729	-0.41309	-0.42851
0.6000	-0.03560	-0.05297	-0.07032	-0.08763	-0.10486	-0.12199	-0.13901
0.5704	0.04116	0.02421	0.00719	-0.00987	-0.02693	-0.04397	-0.06097
0.5000	0.21040	0.19517	0.17968	0.16397	0.14807	0.13199	0.11578
0.4296	0.36620	0.35370	0.34075	0.32740	0.31368	0.29961	0.28516
0.4000	0.42899	0.41794	0.40638	0.39434	0.38186	0.36889	0.35565
0.3000	0.63400	0.62944	0.62415	0.61815	0.61146	0.60412	0.59615
0.2000	0.83841	0.84369	0.84809	0.85161	0.85426	0.85607	0.85703
0.1000	1.06413	1.08608	1.10726	1.12762	1.14712	1.16574	1.18347
0.0500	1.20578	1.24313	1.28019	1.31684	1.35299	1.38855	1.42345
0.0400	1.24028	1.28225	1.32414	1.36584	1.40720	1.44813	1.48852
0.0250	1.30042	1.35153	1.40314	1.45507	1.50712	1.55914	1.61099
0.0200	1.32412	1.37929	1.43529	1.49188	1.54886	1.60604	1.66325
0.0100	1.38267	1.44942	1.51808	1.58838	1.66001	1.73271	1.80621
0.0050	1.42439	1.50114	1.58110	1.66390	1.74919	1.83660	1.92580
0.0020	1.46232	1.55016	1.64305	1.74062	1.84244	1.94806	2.05701
0.0010	1.48216	1.57695	1.67825	1.78572	1.89894	2.01739	2.14053
0.0005	1.49673	1.59738	1.70603	1.82241	1.94611	2.07661	2.21328
0.0001	1.51752	1.62838	1.75053	1.88410	2.02891	2.18448	2.35015

Table 4.13. Frequency Factors (K) for the Log-Pearson Type III Distribution (Cont'd)

	Skew						
Prob.	**-0.6**	**-0.5**	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0.0**
0.9999	-5.04718	-4.82141	-4.59687	-4.37394	-4.15301	-3.93453	-3.71902
0.9995	-4.29311	-4.12443	-3.95605	-3.78820	-3.62113	-3.45513	-3.29053
0.9990	-3.95567	-3.81090	-3.66608	-3.52139	-3.37703	-3.23322	-3.09023
0.9980	-3.60872	-3.48737	-3.36566	-3.24371	-3.12169	-2.99978	-2.87816
0.9950	-3.13232	-3.04102	-2.94900	-2.85636	-2.76321	-2.66965	-2.57583
0.9900	-2.75514	-2.68572	-2.61539	-2.54421	-2.47226	-2.39961	-2.32635
0.9800	-2.35931	-2.31084	-2.26133	-2.21081	-2.15935	-2.10697	-2.05375
0.9750	-2.22702	-2.18505	-2.14202	-2.09795	-2.05290	-2.00688	-1.95996
0.9600	-1.93896	-1.91022	-1.88039	-1.84949	-1.81756	-1.78462	-1.75069
0.9500	-1.79701	-1.77428	-1.75048	-1.72562	-1.69971	-1.67279	-1.64485
0.9000	-1.32850	-1.32309	-1.31671	-1.30936	-1.30105	-1.29178	-1.28155
0.8000	-0.79950	-0.80829	-0.81638	-0.82377	-0.83044	-0.83639	-0.84162
0.7000	-0.44352	-0.45812	-0.47228	-0.48600	-0.49927	-0.51207	-0.52440
0.6000	-0.15589	-0.17261	-0.18916	-0.20552	-0.22168	-0.23763	-0.25335
0.5704	-0.07791	-0.09178	-0.11154	-0.12820	-0.14472	-0.16111	-0.17733
0.5000	0.09945	0.08302	0.06651	0.04993	0.03325	0.01662	0.00000
0.4296	0.27047	0.25558	0.24037	0.22492	0.20925	0.19339	0.17733
0.4000	0.34198	0.32796	0.31362	0.29897	0.28403	0.26882	0.25335
0.3000	0.58757	0.57840	0.56867	0.55839	0.54757	0.53624	0.52440
0.2000	0.85718	0.85653	0.85508	0.85285	0.84986	0.84611	0.84162
0.1000	1.20028	1.21618	1.23114	1.24516	1.25824	1.27037	1.28155
0.0500	1.45762	1.49101	1.52357	1.55527	1.58607	1.61594	1.64485
0.0400	1.52830	1.56740	1.60574	1.64329	1.67999	1.71580	1.75069
0.0250	1.66253	1.71366	1.76427	1.81427	1.86360	1.91219	1.95996
0.0200	1.72033	1.77716	1.83361	1.88959	1.94499	1.99973	2.05375
0.0100	1.88029	1.95472	2.02933	2.10394	2.17840	2.25258	2.32635
0.0050	2.01644	2.10825	2.20092	2.29423	2.38795	2.48187	2.57583
0.0020	2.16884	2.28311	2.39942	2.51741	2.63672	2.75706	2.87816
0.0010	2.26780	2.39867	2.53261	2.66915	2.80786	2.94834	3.09023
0.0005	2.35549	2.50257	2.65390	2.80889	2.96698	3.12767	3.29053
0.0001	2.52507	2.70836	2.89907	3.09631	3.29921	3.50703	3.71902

Table 4.13. Frequency Factors (K) for the Log-Pearson Type III Distribution (Cont'd)

Prob.	Skew						
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**
0.9999	-3.50703	-3.29921	-3.09631	-2.89907	-2.70836	-2.52507	-2.35015
0.9995	-3.12767	-2.96698	-2.80889	-2.65390	-2.50257	-2.35549	-2.21328
0.9990	-2.94834	-2.80786	-2.66915	-2.53261	-2.39867	-2.26780	-2.14053
0.9980	-2.75706	-2.63672	-2.51741	-2.39942	-2.28311	-2.16884	-2.05701
0.9950	-2.48187	-2.38795	-2.29423	-2.20092	-2.10825	-2.01644	-1.92580
0.9900	-2.25258	-2.17840	-2.10394	-2.02933	-1.95472	-1.88029	-1.80621
0.9800	-1.99973	-1.94499	-1.88959	-1.83361	-1.77716	-1.72033	-1.66325
0.9750	-1.91219	-1.86360	-1.81427	-1.76427	-1.71366	-1.66253	-1.61099
0.9600	-1.71580	-1.67999	-1.64329	-1.60574	-1.56740	-1.52830	-1.48852
0.9500	-1.61594	-1.58607	-1.55527	-1.52357	-1.49101	-1.45762	-1.42345
0.9000	-1.27037	-1.25824	-1.24516	-1.23114	-1.21618	-1.20028	-1.18347
0.8000	-0.84611	-0.84986	-0.85285	-0.85508	-0.85653	-0.85718	-0.85703
0.7000	-0.53624	-0.54757	-0.55839	-0.56867	-0.57840	-0.58757	-0.59615
0.6000	-0.26882	-0.28403	-0.29897	-0.31362	-0.32796	-0.34198	-0.35565
0.5704	-0.19339	-0.20925	-0.22492	-0.24037	-0.25558	-0.27047	-0.28516
0.5000	-0.01662	-0.03325	-0.04993	-0.06651	-0.08302	-0.09945	-0.11578
0.4296	0.16111	0.14472	0.12820	0.11154	0.09478	0.07791	0.06097
0.4000	0.23763	0.22168	0.20552	0.18916	0.17261	0.15589	0.13901
0.3000	0.51207	0.49927	0.48600	0.47228	0.45812	0.44352	0.42851
0.2000	0.83639	0.83044	0.82377	0.81638	0.80829	0.79950	0.79002
0.1000	1.29178	1.30105	1.30936	1.31671	1.32309	1.32850	1.33294
0.0500	1.67279	1.69971	1.72562	1.75048	1.77428	1.79701	1.81864
0.0400	1.78462	1.81756	1.84949	1.88039	1.91022	1.93896	1.96660
0.0250	2.00688	2.05290	2.09795	2.14202	2.18505	2.22702	2.26790
0.0200	2.10697	2.15935	2.21081	2.26133	2.31084	2.35931	2.40670
0.0100	2.39961	2.47226	2.54421	2.61539	2.68572	2.75514	2.82359
0.0050	2.66965	2.76321	2.85636	2.94900	3.04102	3.13232	3.22281
0.0020	2.99978	3.12169	3.24371	3.36566	3.48737	3.60872	3.72957
0.0010	3.23322	3.37703	3.52139	3.66608	3.81090	3.95567	4.10022
0.0005	3.45513	3.62113	3.78820	3.95605	4.12443	4.29311	4.46189
0.0001	3.93453	4.15301	4.37394	4.59687	4.82141	5.04718	5.27389

Table 4.13. Frequency Factors (K) for the Log-Pearson Type III Distribution (Cont'd)

	Skew						
Prob.	**0.8**	**0.9**	**1.0**	**1.1**	**1.2**	**1.3**	**1.4**
0.9999	2.18448	-2.02891	-1.88410	-1.75053	-1.62838	-1.51752	-1.41753
0.9995	-2.07661	-1.94611	-1.82241	-1.70603	-1.59738	-1.49673	-1.40413
0.9990	-2.01739	-1.89894	-1.78572	-1.67825	-1.57695	-1.48216	-1.39408
0.9980	-1.94806	-1.84244	-1.74062	-1.64305	-1.55016	-1.46232	-1.37981
0.9950	-1.83660	-1.74919	-1.66390	-1.58110	-1.50114	-1.42439	-1.35114
0.9900	-1.73271	-1.66001	-1.58838	-1.51808	-1.44942	-1.38267	-1.31815
0.9800	-1.60604	-1.54886	-1.49188	-1.43529	-1.37929	-1.32412	-1.26999
0.9750	-1.55914	-1.50712	-1.45507	-1.40314	-1.35153	-1.30042	-1.25004
0.9600	-1.44813	-1.40720	-1.36584	-1.32414	-1.28225	-1.24028	-1.19842
0.9500	-1.38855	-1.35299	-1.31684	-1.28019	-1.24313	-1.20578	-1.16827
0.9000	-1.16574	-1.14712	-1.12762	-1.10726	-1.08608	-1.06413	-1.04144
0.8000	-0.85607	-0.85426	-0.85161	-0.84809	-0.84369	-0.83841	-0.83223
0.7000	-0.60412	-0.61146	-0.61815	-0.62415	-0.62944	-0.63400	-0.63779
0.6000	-0.36889	-0.38186	-0.39434	-0.40638	-0.41794	-0.42899	-0.43949
0.5704	-0.29961	-0.31368	-0.32740	-0.34075	-0.35370	-0.36620	-0.37824
0.5000	-0.13199	-0.14807	-0.16397	-0.17968	-0.19517	-0.21040	-0.22535
0.4296	0.04397	0.02693	0.00987	-0.00719	-0.02421	-0.04116	-0.05803
0.4000	0.12199	0.10486	0.08763	0.07032	0.05297	0.03560	0.01824
0.3000	0.41309	0.39729	0.38111	0.36458	0.34772	0.33054	0.31307
0.2000	0.77986	0.76902	0.75752	0.74537	0.73257	0.71915	0.70512
0.1000	1.33640	1.33889	1.34039	1.34092	1.34047	1.33904	1.33665
0.0500	1.83916	1.85856	1.87683	1.89395	1.90992	1.92472	1.93836
0.0400	1.99311	2.01848	2.04269	2.06573	2.08758	2.10823	2.12768
0.0250	2.30764	2.34623	2.38364	2.41984	2.45482	2.48855	2.52102
0.0200	2.45298	2.49811	2.54206	2.58480	2.62631	2.66657	2.70556
0.0100	2.89101	2.95735	3.02256	3.08660	3.14944	3.21103	3.27134
0.0050	3.31243	3.40109	3.48874	3.57530	3.66073	3.74497	3.82798
0.0020	3.84981	3.96932	4.08802	4.20582	4.32263	4.43839	4.55304
0.0010	4.24439	4.38807	4.53112	4.67344	4.81492	4.95549	5.09505
0.0005	4.63057	4.79899	4.96701	5.13449	5.30130	5.46735	5.63252
0.0001	5.50124	5.72899	5.95691	6.18480	6.41249	6.63980	6.86661

Table 4.13. Frequency Factors (K) for the Log-Pearson Type III Distribution (Cont'd)

Prob.	Skew					
	1.5	**1.6**	**1.7**	**1.8**	**1.9**	**2.0**
0.9999	-1.32774	-1.24728	-1.17520	-1.11054	-1.05239	-0.99990
0.9995	-1.31944	-1.24235	-1.17240	-1.10901	-1.05159	-0.99950
0.9990	-1.31275	-1.23805	-1.16974	-1.10743	-1.50568	-0.99900
0.9980	-1.30279	-1.23132	-1.16534	-1.10465	-1.04898	-0.99800
0.9950	-1.28167	-1.21618	-1.15477	-1.09749	-1.04427	-0.99499
0.9900	-1.25611	-1.19680	-1.14042	-1.08711	-1.03695	-0.98995
0.9800	-1.21716	-1.16584	-1.11628	-1.06864	-1.02311	-0.97980
0.9750	-1.20059	-1.15229	-1.10537	-1.06001	-1.01640	-0.97468
0.9600	-1.15682	-1.11566	-1.07513	-1.03543	-0.99672	-0.95918
0.9500	-1.13075	-1.09338	-1.05631	-1.01973	-0.98381	-0.94871
0.9000	-1.01810	-0.99418	-0.96977	-0.94496	-0.91988	-0.89464
0.8000	-0.82516	-0.81720	-0.80837	-0.79868	-0.78816	-0.77686
0.7000	-0.64080	-0.64300	-0.64436	-0.64488	-0.64453	-0.64333
0.6000	-0.44942	-0.45873	-0.46739	-0.47538	-0.48265	-0.48917
0.5704	-0.38977	-0.40075	-0.41116	-0.42095	-0.43008	-0.43854
0.5000	-0.23996	-0.25422	-0.26808	-0.28150	-0.29443	-0.30685
0.4296	-0.07476	-0.09132	-0.10769	-0.12381	-0.13964	-0.15516
0.4000	0.00092	-0.01631	-0.03344	-0.05040	-0.06718	-0.08371
0.3000	0.29535	0.27740	0.25925	0.24094	0.22250	0.20397
0.2000	0.69050	0.67532	0.65959	0.64335	0.62662	0.60944
0.1000	1.33330	1.32900	1.32376	1.31760	1.31054	1.30259
0.0500	1.95083	1.96213	1.97227	1.98124	1.98906	1.99573
0.0400	2.14591	2.16293	2.17873	2.19332	2.20670	2.21888
0.0250	2.55222	2.58214	2.61076	2.63810	2.66413	2.68888
0.0200	2.74325	2.77964	2.81472	2.84848	2.88091	2.91202
0.0100	3.33035	3.38804	3.44438	3.49935	3.55295	3.60517
0.0050	3.90973	3.99016	4.06926	4.14700	4.22336	4.29832
0.0020	4.66651	4.77875	4.88971	4.99937	5.10768	5.21461
0.0010	5.23353	5.37087	5.50701	5.64190	5.77549	5.90776
0.0005	5.79673	5.95990	6.12196	6.28285	6.44251	6.60090
0.0001	7.09277	7.31818	7.54272	7.76632	7.98888	8.21034

Table 4.14. Calculation of Log-Pearson Type III Discharges for Medina River Using Station Skew

(1) Return Period (yrs)	(2) Exceedence Probability	(3) K	SI Unit (4) Y	SI Unit (5) X (m^3/s)	CU Unit (6) Y	CU Unit (7) X (ft^3/s)
2	0.50	-0.03325	2.078	120	3.626	4,230
5	0.20	0.83044	2.418	262	3.966	9,250
10	0.10	1.30105	2.604	402	4.152	14,200
25	0.04	1.81756	2.807	641	4.355	22,600
50	0.02	2.15935	2.942	875	4.490	30,900
100	0.01	2.47226	3.065	1,160	4.613	41,000

(3) from Table 4.13 for G = 0.2 (rounded from 0.236)

(4) $Y = \overline{Y} + KS_y = 2.091 + 0.394K$

(5) $X = 10^Y$

(6) $Y = \overline{Y} + KS_y = 3.639 + 0.394K$

(7) $X = 10^Y$

Table 4.15. Calculation of Log-Pearson Type III Discharges for Medina River Using Generalized Skew

(1) Return Period (yrs)	(2) Exceedence Probability	(3) K	SI Unit (4) Y	SI Unit (5) X (m^3/s)	CU Unit (6) Y	CU Unit (7) X (ft^3/s)
2	0.50	0.04993	2.111	129	3.659	4,560
5	0.20	0.85285	2.427	267	3.975	9,440
10	0.10	1.24516	2.582	382	4.130	13,500
25	0.04	1.64329	2.738	547	4.286	19,300
50	0.02	1.88959	2.836	685	4.383	24,200
100	0.01	2.10394	2.920	832	4.468	29,400

(3) from Table 4.13 for $\overline{G}$ = -0.3 (rounded from -0.252)

(4) $Y = \overline{Y} + KS_y = 2.091 + 0.394K$

(5) $X = 10^Y$

(6) $Y = \overline{Y} + KS_y = 3.639 + 0.394K$

(7) $X = 10^Y$

Table 4.16. Calculation of Log-Pearson Type III Discharges for Medina River Using Weighted Skew

(1) Return Period (yrs)	(2) Exceedence probability	(3) K	SI Unit		CU Unit	
			(4) Y	(5) X(m³/s)	(6) Y	(7) X (ft³/s)
2	0.50	-0.01662	2.085	121	3.632	4,290
5	0.20	0.83639	2.421	264	3.969	9,310
10	0.10	1.29178	2.600	398	4.148	14,100
25	0.04	1.78462	2.794	622	4.342	22,000
50	0.02	2.10697	2.922	836	4.469	29,400
100	0.01	2.39961	3.036	1,090	4.584	38,400

(3) from Table 4.13 for G_W = 0.1 (rounded from 0.084)

(4) $Y = \overline{Y} + KS_y = 2.091 + 0.394K$

(5) $X = 10^Y$

(6) $Y = \overline{Y} + KS_y = 3.639 + 0.394K$

(7) $X = 10^Y$

4.3.5 Evaluation of Flood Frequency Predictions

The peak flow data for the Medina River gage have now been analyzed by four different frequency distributions and, in the case of log-Pearson Type III distribution, by three different options of skew. The two-parameter log-normal distribution is a special case of the log-Pearson Type III distribution, specifically when the skew is zero. The normal and Gumbel distributions assume fixed skews of zero and 1.139, respectively, for the untransformed data.

The log-Pearson Type III distribution, which uses three parameters, should be superior to all three of the two-parameter distributions discussed in this document. The predicted 10-year and 100-year floods obtained by each of these methods are summarized in Table 4.17. There is considerable variation in the estimates, especially for the 100-year flood, where the values range from 653 m^3/s (23,100 ft^3/s) to 1160 m^3/s (41,000 ft^3/s).

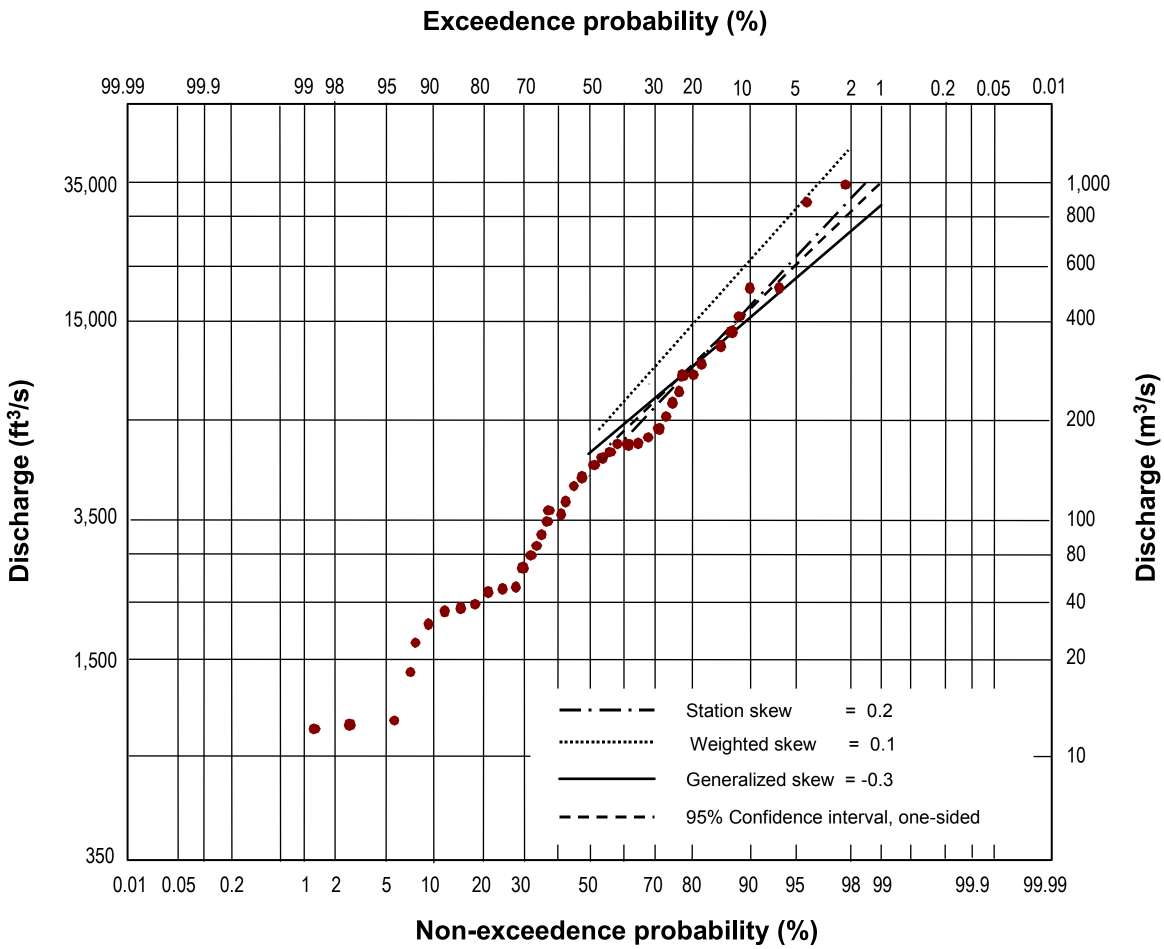

Figure 4.15. Log-Pearson Type III distribution frequency curve, Medina River

The highway designer is faced with the obvious question of which is the appropriate distribution to use for the given set of data. Considerable insight into the nature of the distribution can be obtained by ordering the flood data, computing the mean, standard deviation, and coefficient of skew for the sample and plotting the data on standard probability scales. Based on this preliminary graphical analysis, as well as judgment, some standard distributions might be eliminated before the frequency analysis is begun.

Frequently, more than one distribution or, in the case of the log-Pearson Type III, more than one skew option will seem to fit the data fairly well. Some quantitative measure is needed to determine whether one curve or distribution is better than another. Several different techniques have been proposed for this purpose. Two of the most common are the standard error of estimate and confidence limits, both of which are discussed below.

Table 4.17. Summary of 10- and 100-year Discharges for Selected Probability Distributions

Distribution	Estimated Flow			
	SI (m^3/s)		Customary (ft^3/s)	
	10-yr	100-yr	10-yr	100-yr
Normal	444	653	15,700	23,100
Log-normal	394	1,020	13,900	35,900
Gumbel	485	895	17,100	31,600
Log-Pearson Type III				
Station Skew (G = 0.2)	402	1,160	14,200	41,000
Generalized Skew ($\overline{G}$ = -0.3)	382	832	13,500	29,400
Weighted Skew (G_W = 0.1)	398	1,090	14,100	38,400

4.3.5.1 Standard Error of Estimate

A common measure of statistical reliability is the standard error of estimate or the root-mean square error. Beard (1962) gives the standard error of estimate for the mean, standard deviation, and coefficient of skew as:

$$\text{Mean}: \quad S_{TM} = \frac{S}{n^{0.5}} \tag{4.37}$$

$$\text{Standard Deviation}: \quad S_{TS} = \frac{S}{(2n)^{0.5}} \tag{4.38}$$

$$\text{Coefficient of Skew}: \quad S_{TG} = \left[\frac{6n(n-1)}{(n-2)(n+1)(n+3)}\right]^{0.5} \tag{4.39}$$

These equations show that the standard error of estimate is inversely proportional to the square root of the period of record. In other words, the shorter the record, the larger the standard errors. For example, standard errors for a short record will be approximately twice as large as those for a record four times as long.

The standard error of estimate is actually a measure of the variance that could be expected in a predicted T-year event if the event were estimated from each of a very large number of equally good samples of equal length. Because of its critical dependence on the period of record, the standard error is difficult to interpret, and a large value may be a reflection of a short record.

Using the Medina River annual flood series as an example, the standard errors for the parameters of the log-Pearson Type III computed from Equations 4.37, 4.38, and 4.39 for the logarithms are:

$$S_{TM} = 0.394/(43)^{0.5} = 0.060$$

$$S_{TS} = 0.394/(2(43))^{0.5} = 0.0425$$

$$S_{TG} = [6(43)(42)/((41)(44)(46))]^{0.5} = 0.361$$

The standard error for the skew coefficient of 0.361 is relatively large. The 43-year period of record is statistically of insufficient length to properly evaluate the station skew, and the potential variability in the prediction of the 100-year flood is reflected in the standard error of estimate of the skew coefficient. For this reason, some hydrologists prefer confidence limits for evaluating the reliability of a selected frequency distribution.

4.3.5.2 Confidence Limits

Confidence limits are used to estimate the uncertainties associated with the determination of floods of specified return periods from frequency distributions. Since a given frequency distribution is only a sample estimate of a population, it is probable that another sample taken at the same location and of equal length but taken at a different time would yield a different frequency curve. Confidence limits, or more correctly, confidence intervals, define the range within which these frequency curves could be expected to fall with a specified confidence level.

USGS Bulletin 17B (1982) outlines a method for developing upper and lower confidence intervals. The general forms of the confidence limits are:

$$U_{p,c}(Q) = \overline{Q} + S\,K^{U}_{p,c} \tag{4.40}$$

and

$$L_{p,c}(Q) = \overline{Q} + S\,K^{L}_{p,c} \tag{4.41}$$

where,

- c = level of confidence
- p = exceedence probability
- $U_{p,c}(Q)$ = upper confidence limit corresponding to the values of p and c, for flow Q
- $L_{p,c}(Q)$ = lower confidence limit corresponding to the values of p and c, for flow Q
- $K^{U}_{p,c}$ = upper confidence coefficient corresponding to the values of p and c
- $K^{L}_{p,c}$ = lower confidence coefficient corresponding to the values of p and c

Values of $K^{U}_{p,c}$ and $K^{L}_{p,c}$ for the normal distribution are given in Table 4.18 for the commonly used confidence levels of 0.05 and 0.95. USGS Bulletin 17B (1982), from which Table 4.18 was abstracted, contains a more extensive table covering other confidence levels.

Confidence limits defined in this manner and with the values of Table 4.18 are called one-sided because each defines the limit on just one side of the frequency curve; for 95 percent confidence only one of the values should be computed. The one-sided limits can be combined to form a two-sided confidence interval such that the combination of 95 percent and 5 percent confidence limits define a two-sided 90 percent confidence interval. Practically, this means that at a specified exceedence probability or return period, there is a 5 percent chance the flow will exceed the upper confidence limit and a 5 percent chance the flow will be less than the lower confidence limit. Stated another way, it can be expected that, 90 percent of the time, the specified frequency flow will fall within the two confidence limits.

Table 4.18. Confidence Limit Deviate Values for Normal and Log-normal Distributions (from USGS Bulletin 17B, 1982)

Confidence Level	Systematic Record n	Exceedence Probability								
		0.002	0.010	0.020	0.040	0.100	0.200	0.500	0.800	0.990
0.05	10	4.862	3.981	3.549	3.075	2.355	1.702	0.580	-0.317	-1.563
	15	4.304	3.520	3.136	2.713	2.068	1.482	0.455	-0.406	-1.677
	20	4.033	3.295	2.934	2.534	1.926	1.370	0.387	-0.460	-1.749
	25	3.868	3.158	2.809	2.425	1.838	1.301	0.342	-0.497	-1.801
	30	3.755	3.064	2.724	2.350	1.777	1.252	0.310	-0.525	-1.840
	40	3.608	2.941	2.613	2.251	1.697	1.188	0.266	-0.556	-1.896
	50	3.515	2.862	2.542	2.188	1.646	1.146	0.237	-0.592	-1.936
	60	3.448	2.807	2.492	2.143	1.609	1.116	0.216	-0.612	-1.966
	70	3.399	2.765	2.454	2.110	1.581	1.093	0.199	-0.629	-1.990
	80	3.360	2.733	2.425	2.083	1.559	1.076	0.186	-0.642	-2.010
	90	3.328	2.706	2.400	2.062	1.542	1.061	0.175	-0.652	-2.026
	100	3.301	2.684	2.380	2.044	1.527	1.049	0.166	-0.662	-2.040
0.95	10	1.989	1.563	1.348	1.104	0.712	0.317	-0.580	-1.702	-3.981
	15	2.121	1.677	1.454	1.203	0.802	0.406	-0.455	-1.482	-3.520
	20	2.204	1.749	1.522	1.266	0.858	0.460	-0.387	-1.370	-3.295
	25	2.264	1.801	1.569	1.309	0.898	0.497	-0.342	-1.301	-3.158
	30	2.310	1.840	1.605	1.342	0.928	0.525	-0.310	-1.252	-3.064
	40	2.375	1.896	1.657	1.391	0.970	0.565	-0.266	-1.188	-2.941
	50	2.421	1.936	1.694	1.424	1.000	0.592	-0.237	-1.146	-2.862
	60	2.456	1.966	1.722	1.450	1.022	0.612	-0.216	-1.116	-2.807
	70	2.484	1.990	1.745	1.470	1.040	0.629	-0.199	-1.093	-2.765
	80	2.507	2.010	1.762	1.487	1.054	0.642	-0.186	-1.076	-2.733
	90	2.526	2.026	1.778	1.500	1.066	0.652	-0.175	-1.061	-2.706
	100	2.542	2.040	1.791	1.512	1.077	0.662	-0.166	-1.049	-2.684

When the skew is non-zero, USGS Bulletin 17B (1982) gives the following approximate equations for estimating values of $K^U_{p,c}$ and $K^L_{p,c}$ in terms of the value of $K_{G,p}$ for the given skew and exceedence probability:

$$K^U_{P,C} = \frac{K_{G,P} + \left(K^2_{G,P} - a\,b\right)^{0.5}}{a} \tag{4.42a}$$

and

$$K^L_{P,C} = \frac{K_{G,P} - \left(K^2_{G,P} - a\,b\right)^{0.5}}{a} \tag{4.42b}$$

where

$$a = 1 - \frac{Z_c^2}{2\,(n - 1)} \tag{4.42c}$$

$$b = K^2_{G,P} - \frac{Z_c^2}{n} \tag{4.42d}$$

and where Z_c is the standard normal deviate (zero-skew Pearson Type III deviate) with exceedence probability of (1-c).

Confidence intervals were computed for the Medina River flood series using the USGS Bulletin 17B (1982) procedures for both the log-normal and the log-Pearson Type III distributions. The weighted skew of 0.1 was used with the log-Pearson Type III analysis. The computations for the confidence intervals are given in Tables 4.19 (log-normal) and 4.20 (log-Pearson Type III). The confidence intervals for the log-normal and log-Pearson Type III are shown in Figures 4.13 and 4.15, respectively.

It appears that a log-Pearson Type III would be the most acceptable distribution for the Medina River data. The actual data follow the distribution very well, and all the data fall within the confidence intervals. Based on this analysis, the log-Pearson Type III would be the preferred standard distribution with the log-normal also acceptable. The normal and Gumbel distributions are unsatisfactory for this particular set of data.

Table 4.19. Computation of One-sided, 95 Percent Confidence Interval for the Log-normal Analysis of the Medina River Annual Maximum Series

			SI			CU		
(1) Return Period (yrs)	(2) Exceedence Probability	(3) K^u	(4) U	(5) X^u (m^3/s)	(6) X (m^3/s)	(7) U	(8) X^u (ft^3/s)	(9) X (ft^3/s)
2	0.5	0.2573	2.192	156	123	3.740	5,500	4,360
5	0.2	1.1754	2.554	358	265	4.102	12,600	9,350
10	0.1	1.6817	2.754	568	394	4.302	20,000	13,900
25	0.04	2.2321	2.970	935	604	4.518	33,000	21,300
50	0.02	2.5917	3.112	1,300	795	4.660	45,700	28,100
100	0.01	2.9173	3.241	1,740	1,020	4.788	61,400	35,900
500	0.002	3.5801	3.502	3,180	1,680	5.050	112,200	59,300

(3) interpolated from Table 4.18 for a record length of 43 years

(4) $U = \overline{Y} + S_y K^U = 2.091 + 0.394\,K^U$

(5) $X^U = 10^U$

(6) estimated using Equations 4.29 and 4.30

(7) $U = \overline{Y} + S_y K^U = 3.639 + 0.394\ K^U$
(8) $X^U = 10^U$
(9) estimated using Equations 4.29 and 4.30

Table 4.20. Computation of One-sided, 95 Percent Confidence Interval for the Log-Pearson Type III Analysis of the Medina River Annual Maximum Series with Weighted Skew

					SI			CU		
(1) Return Period (yrs)	(2) Exceedence Probability	(3) K	(4) b	(5) K^U	(6) U	(7) X^U (m^3/s)	(8) X (m^3/s)	(9) U	(10) X^U (ft^3/s)	(11) X (ft^3/s)
2	0.5	-0.01662	-0.0627	0.2378	2.185	153	121	3.733	5,410	4,290
5	0.2	0.83639	0.6366	1.1627	2.549	354	264	4.097	12,500	9,310
10	0.1	1.29178	1.6058	1.6847	2.755	569	398	4.303	20,090	14,060
25	0.04	1.78462	3.1219	2.2618	2.982	959	622	4.530	33,880	21,980
50	0.02	2.10697	4.3764	2.6437	3.133	1,360	834	4.681	47,970	29,440
100	0.01	2.39961	5.6952	2.9924	3.270	1,860	1,090	4.818	65,770	38,370
500	0.002	2.99978	8.9357	3.7116	3.553	3,570	1,870	5.101	126,180	66,220

(3) from Table 4.13 for skew *G* = 0.1

(4) from Equation 4.42d

$$b = K^2 - \frac{Z_c^2}{n} = K^2 - \frac{(1.645)^2}{43} = K^2 - 0.06293$$

(5) from Equation 4.42a

$$K^U = \frac{K + (K^2 - ab)^{0.5}}{a} = \frac{K + (K^2 - 0.96779\ b)^{0.5}}{0.96779}$$

(6) from Equation 4.40

$$U = \overline{Y} + S_Y K^U = 2.091 + 0.394 K^U$$

(7) from Equation 4.35

$$X^U = 10^U$$

(8) from Table 4.16

(9) from Equation 4.40

$$U = \overline{Y} + S_Y K^U = 3.639 + 0.394 K^U$$

(10) from Equation 4.35

$$X^U = 10^U$$

(11) from Table 4.16

4.3.6 Other Considerations in Frequency Analysis

In the course of performing frequency analyses for various watersheds, the designer will undoubtedly encounter situations where further adjustments to the data are indicated. Additional analysis may be necessary due to outliers, inclusion of historical data, incomplete records or years with zero flow, and mixed populations. Some of the more common methods of analysis are discussed in the following paragraphs.

4.3.6.1 Outliers

Outliers, which may be found at either or both ends of a frequency distribution, are measured values that occur, but appear to be from a longer sample or different population. This is reflected when one or more data points do not follow the trend of the remaining data.

USGS Bulletin 17B (1982) presents criteria based on a one-sided test to detect outliers at a 10 percent significance level. If the station skew is greater than 0.4, tests are applied for high outliers first, and, if less than -0.4, low outliers are considered first. If the station skew is between ± 0.4, both high and low outliers are tested before any data are eliminated. The detection of high and low outliers is obtained with the following equations, respectively:

$$Y_H = \bar{Y} + K_N S_Y \tag{4.43}$$

and

$$Y_L = \bar{Y} - K_N S_Y \tag{4.44}$$

where,

Y_H, Y_L = log of the high or low outlier limit, respectively
$\bar{Y}$ = mean of the log of the sample flows
S_y = standard deviation of the sample
K_N = critical deviate (from Table 4.21).

If the sample is found to contain high outliers, the peak flows should be checked against other historical data sources and data from nearby stations. This check enables categorization of the flow observation as a potential anomaly or error in the sample. USGS Bulletin 17B (1982) recommends that high outliers be adjusted for historical information or retained in the sample as a systematic peak. The high outlier should not be discarded unless the peak flow is shown to be seriously in error. If a high outlier is adjusted based on historical data, the mean and standard deviation of the log distribution should be recomputed for the adjusted data before testing for low outliers.

To test for low outliers, the low outlier threshold Y_L of Equation 4.44 is computed. The corresponding discharge $X_L = 10^{Y_L}$ is then computed. If any discharges in the flood series are less than X_L, then they are considered to be low outliers and should be deleted from the sample. The moments should be recomputed and the conditional probability adjustment from the arid lands hydrology section of Chapter 9 (Special Topics) applied.

Table 4.21. Outlier Test Deviates (K_N) at 10 Percent Significance Level (from USGS Bulletin 17B, 1982)

Sample Size	K_N Value	Sample Size	K_N Value	Sample Size	K_N Value	Sample Size	K_N Value
10	2.036	45	2.727	80	2.940	115	3.064
11	2.088	46	2.736	81	2.945	116	3.067
12	2.134	47	2.744	82	2.949	117	3.070
13	2.165	48	2.753	83	2.953	118	3.073
14	2.213	49	2.760	84	2.957	119	3.075
15	2.247	50	2.768	85	2.961	120	3.078
16	2.279	51	2.775	86	2.966	121	3.081
17	2.309	52	2.783	87	2.970	122	3.083
18	2.335	53	2.790	88	2.973	123	3.086
19	2.361	54	2.798	89	2.977	124	3.089
20	2.385	55	2.804	90	2.989	125	3.092
21	2.408	56	2.811	91	2.984	126	3.095
22	2.429	57	2.818	92	2.889	127	3.097
23	2.448	58	2.824	93	2.993	128	3.100
24	2.467	59	2.831	94	2.996	129	3.102
25	2.487	60	2.837	95	3.000	130	3.104
26	2.502	61	2.842	96	3.003	131	3.107
27	2.510	62	2.849	97	3.006	132	3.109
28	2.534	63	2.854	98	3.011	133	3.112
29	2.549	64	2.860	99	3.014	134	3.114
30	2.563	65	2.866	100	3.017	135	3.116
31	2.577	66	2.871	101	3.021	136	3.119
32	2.591	67	2.877	102	3.024	137	3.122
33	2.604	68	2.883	103	3.027	138	3.124
34	2.616	69	2.888	104	3.030	139	3.126
35	2.628	70	2.893	105	3.033	140	3.129
36	2.639	71	2.897	106	3.037	141	3.131
37	2.650	72	2.903	107	3.040	142	3.133
38	2.661	73	2.908	108	3.043	143	3.135
39	2.671	74	2.912	109	3.046	144	3.138
40	2.682	75	2.917	110	3.049	145	3.140
41	2.692	76	2.922	111	3.052	146	3.142
42	2.700	77	2.927	112	3.055	147	3.144
43	2.710	78	2.931	113	3.058	148	3.146
44	2.720	79	2.935	114	3.061	149	3.148

Example 4.10. To illustrate these criteria for outlier detection, Equations 4.43 and 4.44 are applied to the 43-year record for the Medina River, which has a log mean of 2.091 (3.639 in CU units) and a log standard deviation of 0.394. From Table 4.21, K_N = 2.710.

Testing first for high outliers:

Variable	**Value in SI**	**Value in CU**
Y_H	$2.091 + 2.710\,(0.394) = 3.159$	$3.639 + 2.710\,(0.394) = 4.707$
X_H	$10^{3.159} = 1{,}440\ m^3/s$	$10^{4.707} = 50{,}900\ ft^3/s$

No flows in the sample exceed this amount, so there are no high outliers. Testing for low outliers, Equation 4.44 gives:

Variable	**Value in SI**	**Value in CU**
Y_L	$2.091 - 2.710\,(0.394) = 1.023$	$3.639 - 2.710\,(0.394) = 2.571$
X_L	$10^{1.023} = 11\ m^3/s$	$10^{2.571} = 372\ ft^3/s$

There are no flows in the Medina River sample that are less than this critical value. Therefore, the entire sample should be used in the log-Pearson Type III analysis.

4.3.6.2 Historical Data

When reliable information indicates that one or more large floods occurred outside the period of record, the frequency analysis should be adjusted to account for these events. Although estimates of unrecorded historical flood discharges may be inaccurate, they should be incorporated into the sample because the error in estimating the flow is small in relation to the random variability in the peak flows from year to year. If, however, there is evidence these floods resulted under different watershed conditions or from situations that differ from the sample, the large floods should be adjusted to reflect current watershed conditions.

USGS Bulletin 17B (1982) provides methods to adjust for historical data based on the assumption that "the data from the systematic (station) record is representative of the intervening period between the systematic and historic record lengths." Two sets of equations for this adjustment are given in Bulletin 17B. The first is applied directly to the log-transformed station data, including the historical events. The floods are reordered, assigning the largest historic flood a rank of one. The order number is then weighted giving a weight of 1.00 to the historic event, and weighting the order of the station data by a value determined from the equation:

$$W = \frac{H - Z}{n + L} \qquad (4.45)$$

where,

W = the weighting factor

H = the length of the historic period of years
Z = the number of historical events included in the analysis
L = the number of low outliers excluded from the analysis.

The properties of the historically extended sample are then computed according to the equations

$$\overline{Q}_L{}' = \frac{W \sum Q_L + \sum Q_{L,Z}}{H - WL} \tag{4.46}$$

$$(S_L{}')^2 = \frac{W \sum (Q_L - Q_L{}')^2 + \sum (Q_{L,Z} - \overline{Q}_L{}')^2}{H - WL - 1} \tag{4.47}$$

and

$$G_L{}' = \frac{H - WL}{(H - WL - 1)(H - WL - 2)} \left[\frac{W \sum (Q_L - \overline{Q}_L{}')^3 + \sum (Q_{L,Z} - \overline{Q}_L{}')^3}{(S_L{}')^3} \right] \tag{4.48}$$

where,
- $\overline{Q}_L'$ = historically adjusted mean log transform of the flows
- Q_L = log transform of the flows contained in the sample record
- $Q_{L,Z}$ = log of the historic peak flow
- S_L' = historically adjusted standard deviation
- G_L' = historically adjusted skew coefficient.

All other values are as previously defined. In the case where the sample properties were previously computed such as were done for the Medina River, USGS Bulletin 17B (1982) gives the following adjustments that can be applied directly

$$\overline{Q}_L{}' = \frac{W n \overline{Q}_L + \sum Q_{L,Z}}{H - WL} \tag{4.49}$$

$$(S_L{}')^2 = \frac{W(n-1) S_L^2 + W n (\overline{Q}_L - \overline{Q}_L{}')^2 + \sum (Q_{L,Z} - \overline{Q}_L{}')^2}{H - WL - 1} \tag{4.50}$$

$$G_L{}' = \frac{H - WL}{(H - WL - 1)(H - WL - 2)(S_L{}')^3} \times \left[\frac{W(n-1)(n-2) S_L^3 G_L}{n} + 3W(n-1)(\overline{Q}_L - \overline{Q}_L{}') S_L^2 + Wn(\overline{Q}_L - \overline{Q}_L{}')^3 + \sum (Q_{L,Z} - \overline{Q}_L{}')^3 \right] \tag{4.51}$$

Once the adjusted statistical parameters are determined, the log-Pearson Type III distribution is determined by Equation 4.27 using the Weibull plotting position formula:

$$P = \frac{m'}{H + 1} \tag{4.52}$$

where m´ is the adjusted rank order number of the floods including historical events, where

$$m' = m \qquad \text{for } 1 \leq m \leq Z$$

$$m' = Wm - (W - 1)(Z + 0.5) \qquad \text{for } (Z + 1) \leq m \leq (Z + nL)$$

Detailed examples illustrating the computations for the historic adjustment are contained in USGS Bulletin 17B (1982) and the designer should consult this reference for further information.

4.3.6.3 Incomplete Records and Zero Flows

Stream flow records are often interrupted for a variety of reasons. Gages may be removed for some period of time, there may be periods of zero flow that are common in the arid regions of the United States, and there may be periods when a gage is inoperative either because the flow is too low to record or it is too large and causes a gage malfunction.

If the break in the record is not flood related, such as the removal of a gage, no special adjustments are needed and the segments of the interrupted record can be combined together to produce a record equal to the sum of the length of the segments. When a gage malfunctions during a flood, it is usually possible to estimate the peak discharge from highwater marks or slope-area calculations. The estimate is made a part of the record, and a frequency analysis performed without further adjustment.

Zero flows or flows that are too low to be recorded present more of a problem because, in the log transform, these flows produce undefined values. In this case, USGS Bulletin 17B (1982) presents an adjustment based on conditional probability that is applicable if not more than 25 percent of the sample is eliminated.

The adjustment for zero flows also is applied only after all other data adjustments have been made. The adjustment is made by first calculating the relative frequency, P_a, that the annual peak will exceed the level below where either flows are zero or not considered (the truncation level):

$$P_a = \frac{M}{n} \tag{4.53}$$

where M is the number of flows above the truncated level and n is the total period of record. The exceedence probabilities, P, of selected points on the frequency curve are recomputed as a conditional probability as follows

$$P = P_a \, P_d \tag{4.54}$$

where P_d is the selected probability.

Since the frequency curve adjusted by Equation 4.54 has unknown statistics, its properties, synthetic values, are computed by the equations:

$$\overline{Q}_s = \log(Q_{0.50}) - K_{0.50}(S_S) \tag{4.55}$$

$$S_S = \frac{\log(Q_{0.01}/Q_{0.50})}{K_{0.01} - K_{0.50}} \tag{4.56}$$

and

$$G_S = -2.50 + 3.12\left[\frac{\log(Q_{0.01}/Q_{0.10})}{\log(Q_{0.10}/Q_{0.50})}\right] \tag{4.57}$$

where $\overline{Q}_s$, S_s, and G_s are the mean, standard deviation, and skew of the synthetic frequency curve, $Q_{0.01}$, $Q_{0.10}$, and $Q_{0.50}$ are discharges with exceedence probabilities of 0.01, 0.10 and 0.50, respectively, and $K_{0.01}$ and $K_{0.50}$ are the log-Pearson Type III deviates for exceedence probabilities of 0.01 and 0.50, respectively. The values of $Q_{0.01}$, $Q_{0.10}$ and $Q_{0.50}$ must usually be interpolated since probabilities computed with Equation 4.53 are not normally those needed to compute the properties of the synthetic or truncated distribution.

The log-Pearson Type III distribution can then be computed in the conventional manner using the synthetic statistical properties. USGS Bulletin 17B (1982) recommends the distribution be compared with the observed flows since data adjusted for conditional probability may not follow a log-Pearson Type III distribution.

4.3.6.4 Mixed Populations

In some areas of the United States, floods are caused by combinations of events (e.g., rainfall and snowmelt in mountainous areas or rainfall and hurricane events along the Gulf and Atlantic coasts). Records from such combined events are said to be mixed populations. These records are often characterized by very large skew coefficients and, when plotted, suggest that two different distributions might be applicable.

Such records should be divided into two separate records according to their respective causes, with each record analyzed separately by an appropriate frequency distribution. The two separate frequency curves can then be combined through the concept of the addition of the probabilities of two events as follows:

$$Pr(Q \text{ or } Q_m) = Pr(Q) + Pr(Q_m) - Pr(Q)Pr(Q_m) \tag{4.58}$$

4.3.6.5 Two-Station Comparison

The objective of this method is to improve the mean and standard deviation of the logarithms at a short-record station (Y) using the statistics from a nearby long-record station (X). The method is from Appendix 7 of USGS Bulletin 17B (1982). The steps of the procedure depend on the nature of the records. Specifically, there are two cases: (1) the entire short record occurred during the duration of the long-record station, and (2) only part of the short record occurred during the duration of the long-record station. The following notation applies to the procedure:

N_x = record length at long-record station

N_1 = number of years when flows were concurrently observed at X and Y