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**"Using Spreadsheets for Calculating
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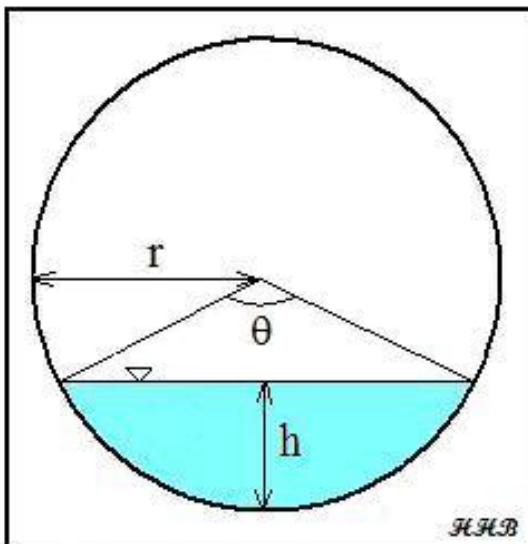
Spreadsheet Use for Partially Full Pipe Flow Calculations

Harlan H. Bengtson, PhD, P.E.

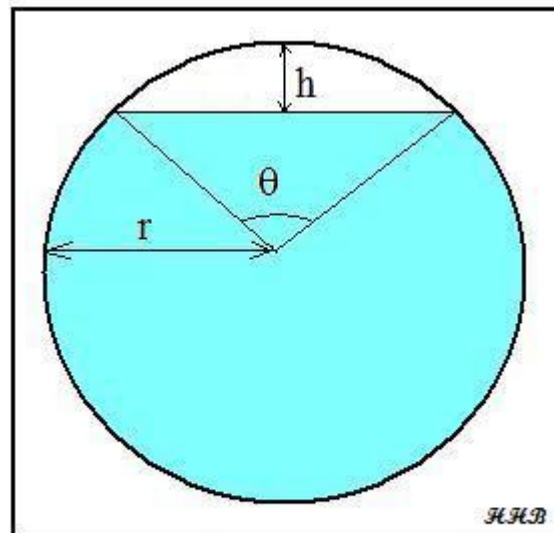
COURSE CONTENT

1. Introduction

The Manning equation can be used for uniform flow in a pipe, but the Manning roughness coefficient needs to be considered to be variable, dependent upon the depth of flow. This course includes a review of the Manning equation, along with presentation of equations for calculating the cross-sectional area, wetted perimeter, and hydraulic radius for flow of a specified depth in a pipe of known diameter. Equations are also given for calculating the Manning roughness coefficient, n , for a given depth of flow in a pipe of known diameter. Numerous worked examples illustrate the use of these equations together with the Manning equation for partially full pipe flow. A spreadsheet for making partially full pipe flow calculations is included with this course and its use is discussed and illustrated through worked examples.



Partially Full Pipe Flow Parameters
(Less Than Half Full)



Partially Full Pipe Flow Parameters
(More Than Half Full)

2. Learning Objectives

At the conclusion of this course, the student will

- Be able to calculate the cross-sectional area of flow, wetted perimeter, and hydraulic radius for less than half full flow at a given depth in a pipe of given diameter.
- Be able to calculate the cross-sectional area of flow, wetted perimeter, and hydraulic radius for more than half full flow at a given depth in a pipe of given diameter.
- Be able to use Figure 3 in the course material to determine the flow rate at a given depth of flow in a pipe of known diameter if the full pipe flow rate is known or can be calculated.
- Be able to use Figure 3 the course material to determine the average water velocity at a given depth of flow in a pipe of known diameter if the full pipe average velocity is known or can be calculated.
- Be able to calculate the Manning roughness coefficient for a given depth of flow in a pipe of known diameter, with a known Manning roughness coefficient for full pipe flow.
- Be able to use the Manning equation to calculate the flow rate and average velocity for flow at a specified depth in a pipe of specified diameter, with known pipe slope and full pipe Manning roughness coefficient.
- Be able to calculate the normal depth for a specified flow rate of water through a pipe of known diameter, slope, and full pipe Manning roughness coefficient

- Be able to carry out the calculations in the above learning objectives using either U.S. units or S.I. units.
- Be able to use the spreadsheet included with this course to make partially full pipe flow calculations.

3. Topics Covered in this Course

- I. Manning Equation Review
- II. Hydraulic Radius - Less than Half Full Flow
- III. Hydraulic Radius - More than Half Full Flow
- IV. Use of Variable n in the Manning Equation
- V. Equations for Variable Manning roughness coefficient
- VI. Flow Rate Calculation for Less than Half Full Flow
- VII. Flow Rate Calculation for More than Half Full Flow
- VIII. Normal Depth Calculation Review
- IX. Normal Depth for Less than Half Full Flow
- X. Normal Depth for More than Half Full Flow
- XI. Summary
- XII. References

4. Manning Equation Review

The most widely used equation for uniform open channel flow* calculations is the Manning equation:

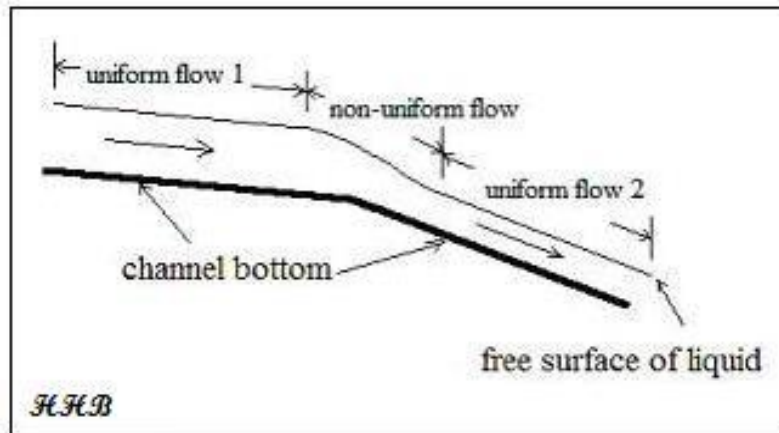
$$Q = (1.49/n)A(R_h^{2/3})S^{1/2} \quad (1)$$

Where:

- **Q** is the volumetric flow rate passing through the channel reach in cfs.
- **A** is the cross-sectional area of flow normal to the flow direction in ft².
- **S** is the bottom slope of the channel** in ft/ft (dimensionless).
- **n** is a dimensionless empirical constant called the Manning Roughness coefficient.
- **R_h** is the hydraulic radius = A/P.
- **P** is the wetted perimeter of the cross-sectional area of flow in ft.

*You may recall that uniform open channel flow (which is required for use of the Manning equation) occurs for a constant flow rate of water through a channel with constant slope, size and shape, and roughness. Uniform and non-uniform flow are illustrated in the diagram below.

Uniform partially full pipe flow occurs for a constant flow rate of water through a pipe of constant diameter, surface roughness, and slope. Under these conditions the water will flow at a constant depth.



**S is actually the slope of the hydraulic grade line. For uniform flow, the depth of flow is constant, so the slope of the hydraulic grade line is the same as the slope of the liquid surface and the same as the channel bottom slope. The channel bottom slope is typically used for S in the Manning equation.

It should also be noted that the Manning equation is a dimensional equation. With the 1.49 constant in equation (1), the parameters in the equation must have the units shown in the list below the equation.

For S.I. units, the constant in the Manning equation changes slightly to the following:

$$Q = (1.00/n)A(R_h^{2/3})S^{1/2} \quad (2)$$

Where:

- Q is the volumetric flow rate passing through the channel reach in m³s.
- A is the cross-sectional area of flow normal to the flow direction in m².

- **S** is the bottom slope of the channel in m/m (dimensionless).
- **n** is a dimensionless empirical constant called the Manning Roughness coefficient.
- **R_h** is the hydraulic radius = A/P .
- **P** is the wetted perimeter of the cross-sectional area of flow in m.

Table 1. Typical Manning Roughness Coefficient Values

<u>Channel Surface</u>	<u>Manning Roughness Coefficient, n</u>
Asbestos cement	0.011
Brass	0.011
Brick	0.015
Cast-iron, new	0.012
Concrete, steel forms	0.011
Concrete, wooden forms	0.015
Concrete, centrifugally spun	0.013
Copper	0.011
Corrugated metal	0.022
Galvanized Iron	0.016
Lead	0.011
Plastic	0.009
Steel - Coal-tar enamel	0.01
Steel - New unlined	0.011
Steel - Riveted	0.019
Wood stave	0.012

Values of the Manning roughness coefficient, n , for some common open channel materials are given in Table 1 above. The source for the n values in the table is www.engineeringtoolbox.com.

5. Hydraulic Radius – Less than Half Full Flow

The hydraulic radius is one of the parameters needed for Manning equation calculations. Equations are available to calculate the hydraulic radius for known pipe diameter and depth of flow. The equations are slightly different depending on whether the pipe is flowing less than half or more than half full. The calculations for less than half full pipe flow will be covered in this section and the more than half full calculation will be covered in the next section.

The equations needed to calculate the cross sectional area of flow, A , the wetted perimeter, P , and the hydraulic radius, R_h , are shown below, along with a diagram showing the parameters for a pipe flowing less than half full. Note that the parameters r and h are used in the equations for A and P . For this case of less than half full flow, h is simply equal to the depth of flow y , while r is the radius of the pipe, which is $D/2$.

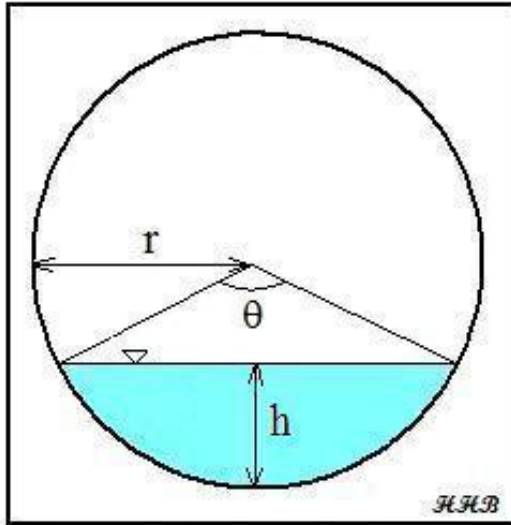


Figure 1. Partially Full Pipe Flow Parameters (Less than Half Full)

$$r = \frac{D}{2} \quad h = y$$

$$\theta = 2 \arccos \left(\frac{r-h}{r} \right)$$

$$A = \frac{r^2(\theta - \sin \theta)}{2}$$

$$P = r \theta$$

$$R_h = A/P$$

Example #1: Calculate the hydraulic radius (in ft) for water flowing 6 inches deep in a 48 inch diameter storm sewer.

Solution: $r = D/2 = 24 \text{ in} = 2 \text{ ft}$; $h = y = 6 \text{ in} = 0.5 \text{ ft}$;

$$\theta = 2 \arccos [(2 - 0.5)/2] = 1.45 \text{ radians}$$

$$A = [2^2 (1.45 - \sin (1.45))] / 2 = 0.91 \text{ ft}^2$$

$$P = (2)(1.45) = 2.9 \text{ ft}$$

$$R_h = 0.91/2.9 = \underline{\underline{0.31 \text{ ft}}}$$

Partially Full Pipe Flow Calculations - U.S. Units					
I. Calculation of Discharge, Q, and average velocity, V					
for pipes less than half full					
Instructions: Enter values in blue boxes. Spreadsheet calculates values in yellow boxes					
Inputs			Calculations		
Pipe Diameter, D =	48	in	Pipe Diameter, D =	4	ft
Depth of flow, y =	0.5	ft	Pipe Radius, r =	2	ft
(must have $y \leq D/2$)					
Manning			Circ. Segment Height, h =	0.5	ft
roughness, n_{full} =	0.012		Central Angle, θ =	1.45	radians
Channel bottom			Cross-Sect. Area, A =	0.91	ft ²
slope, S =	0.0003	ft/ft	Wetted Perimeter, P =	2.9	ft
y/D =	0.125		Hydraulic Radius, R =	0.31	ft

The screenshot above shows part of the “Q_less than half full” worksheet in the spreadsheet that was included with this course. It shows the solution to Example #1. All that is necessary is entry of the pipe diameter and the depth of flow. The spreadsheet will then calculate the area of flow, wetted perimeter, and hydraulic radius.

Example #2: Calculate the hydraulic radius (in m) for water flowing 20 mm deep in a pipe of 100 mm diameter.

Solution: $r = D/2 = 50 \text{ mm} = 0.050 \text{ m}$; $h = y = 20 \text{ mm} = 0.020 \text{ m}$;

$$\theta = 2 \arccos [(0.050 - 0.020)/0.050] = 1.85 \text{ radians}$$

$$A = [0.05^2 (1.85 - \sin (1.85))] / 2 = 0.00111 \text{ m}^2$$

$$P = (0.05)(1.85) = 0.0925 \text{ m}$$

$$R_h = 0.00111/0.0925 = \underline{\underline{0.0120 \text{ m}}}$$

6. Hydraulic Radius – More than Half Full Flow

The equations for calculating the cross-sectional area of flow, A , the wetted perimeter, P , and the hydraulic radius, R_h , are shown below alongside a diagram showing the parameters in the equations. For more than half full pipe flow, the parameter h is $2r - y$, instead of simply being equal to y as for less than half full pipe flow.

Calculation of the area of flow and the wetted perimeter are slightly different than those calculations for the less than half full case. The area of flow is calculated as the total cross-sectional area of the pipe minus the cross-sectional area of the empty space above the water. Similarly the wetted perimeter is calculated as the total perimeter minus the dry perimeter at the top of the pipe. These equations are shown below along with a diagram for “more than half full” pipe flow.

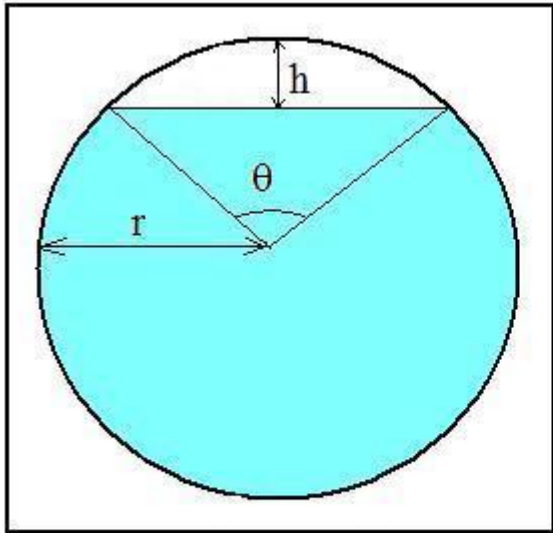


Figure 2. Partially Full Pipe Flow Parameters (more than half full)

$$r = D/2 \quad h = 2r - y$$

$$\theta = 2 \arccos \left(\frac{r-h}{r} \right)$$

$$A = \pi r^2 - \frac{r^2(\theta - \sin \theta)}{2}$$

$$P = 2\pi r - r\theta$$

$$R_h = A/P$$

Example #3: Calculate the hydraulic radius for water flowing 3.4 ft deep in a 48 inch diameter storm sewer.

Solution: $r = 48/2 = 24 \text{ inches} = 2 \text{ ft}$; $h = 2*2 - 3.4 = 0.6 \text{ ft}$

$$\theta = 2 \arccos [(2 - 0.6)/2] = 1.59 \text{ radians}$$

$$A = \pi (2^2) - [2^2(1.59 - \sin (1.59))] / 2 = 11.38 \text{ ft}^2$$

$$P = 2 \pi (2) - (2)(1.59) = 9.4 \text{ ft}$$

$$R_h = 11.38/9.4 = \underline{\underline{1.21 \text{ ft}}}$$

This example can also be solved with the course spreadsheet as illustrated in the screenshot below, which is from the “Q_more than half full” tab in the course spreadsheet. As you can see, the values for **A**, **P**, and **R_h** are the same as in the calculations above.

Partially Full Pipe Flow Calculations - U.S. Units							
II. Calculation of Discharge, Q, and average velocity, V							
for pipes more than half full							
Instructions: Enter values in blue boxes. Spreadsheet calculates values in yellow boxes							
Inputs				Calculations			
Pipe Diameter, D =	48	in		Pipe Diameter, D =	4	ft	
Depth of flow, y =	3.4	ft		Pipe Radius, r =	2	ft	
(must have $y \geq D/2$)							
Full Pipe Manning				Circ. Segment Height, h =	0.6	ft	
roughness, n_{full} =	0.012			Central Angle, θ =	1.59	radians	
Channel bottom				Cross-Sect. Area, A =	11.38	ft ²	
slope, S =	0.0003	ft/ft		Wetted Perimeter, P =	9.4	ft	
Calculations				Hydraulic Radius, R =	1.21	ft	

7. Use of Variable **n** in the Manning Equation

The cross-sectional area, **A**; wetted perimeter, **P**; and hydraulic radius, **R_h**; can be calculated using the geometric/trigonometric equations presented in the previous two sections. It thus seems logical that the **A** and **R_h** values calculated in this manner could be used in the Manning equation (along with the pipe slope, and the

Manning roughness coefficient value for full pipe flow) to calculate flow rate for given depth of flow or normal depth for given flow rate in partially full pipe flow.

Unfortunately, as early as the mid twentieth century, it had been observed that measured flow rates in partially full pipe flow do not agree with values calculated as just described above. T. R. Camp developed a method for improving the agreement between measured values of partially full pipe flow rate and values calculated with the Manning equation. He did this by using a variation in Manning roughness coefficient with depth of flow in the pipe as a fraction of the pipe diameter. That is, he used a variation in n/n_{full} as a function of y/D . His procedure is described in his 1946 article, "Design of Sewers to Facilitate Flow," which is reference #3 at the end of this course. T. R. Camp's work led to the graph below, which shows the variation of Q/Q_{full} , V/V_{full} , and n/n_{full} as functions of the ratio of depth of flow to pipe diameter (y/D).

The graph developed by Camp and shown in the diagram below appears in several publications of the American Society of Civil Engineers, the Water Pollution Control Federation, and the Water Environment Federation from 1969 through 1992, as well as in many environmental engineering textbooks. The graph below was prepared from values read off a similar graph in Steel and McGhee's textbook (reference #5 at the end of this course).

Prior to the common use of spreadsheets, which make calculations with the trigonometric/geometric equations for A , P , and R_h , relatively easy, use of the graph below was a widely used method of handling partially full pipe flow calculations. V_{full} and Q_{full} can be calculated for full pipe flow conditions in a given pipe with the Manning equation. Then V and Q can be found for any depth of flow, y , in that pipe by reading values off the graph.

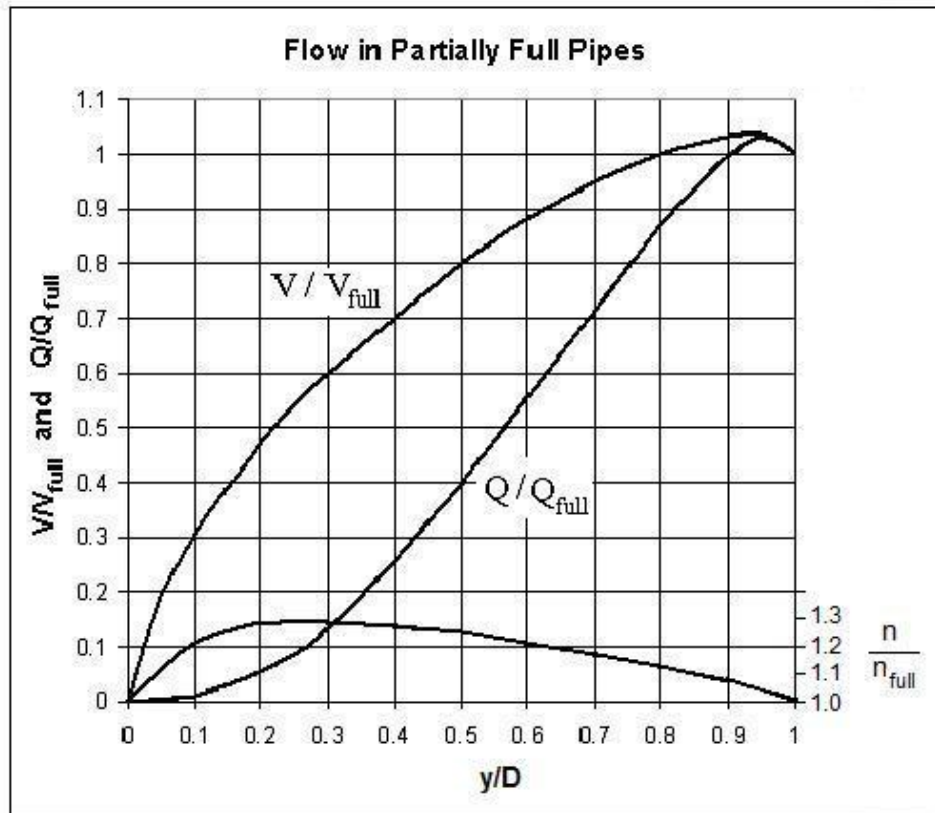


Figure 3. Flow in Partially Full Pipes

Although the variation in Manning roughness coefficient, n , shown in the graph above, doesn't make sense intuitively, it does work well in calculating values of flow rate, velocity, or normal depth that agree with empirical measurements. Keep in mind that the Manning equation was developed for flow in open channels with rectangular, trapezoidal, and similar cross-sections. It works very well for those channel shapes with a constant value for the Manning roughness coefficient, n . For partially full pipe flow, however, using the variation in n with depth of flow as proposed by Camp is a preferred method.

Example #4: The flow rate and average velocity in a particular 21 inch diameter storm sewer when it is flowing full, have been calculated to be: $Q_{full} = 9.12$ cfs and

$V_{full} = 3.79$ ft/sec. Estimate the average velocity and flow rate in this storm sewer when it is flowing

a) at a depth of 8.4 inches and

b) at a depth of 14.7 inches.

Solution: a) The depth/diameter ratio is: $y/D = 8.4/21 = 0.4$. From the “Flow in Partially Full Pipes” graph above, at $y/D = 0.4$: $V/V_{full} = 0.7$ and $Q/Q_{full} = 0.25$. The flow rate and average velocity at $y = 8.4$ inches can now be calculated as follows:

$$V = (V/V_{full})(V_{full}) = (0.7)(3.79) \text{ ft/sec} = \underline{\underline{1.95 \text{ ft/sec}}}$$

$$Q = (Q/Q_{full})(Q_{full}) = (0.25)(9.12) \text{ cfs} = \underline{\underline{2.28 \text{ cfs}}}$$

b) The depth/diameter ratio is: $y/D = 14.7/21 = 0.7$. From the “Flow in Partially Full Pipes” graph above, at $y/D = 0.7$: $V/V_{full} = 0.95$ and $Q/Q_{full} = 0.7$. The flow rate and average velocity at $y = 14.7$ inches can now be calculated as follows:

$$V = (V/V_{full})(V_{full}) = (0.95)(3.79) \text{ ft/sec} = \underline{\underline{2.65 \text{ ft/sec}}}$$

$$Q = (Q/Q_{full})(Q_{full}) = (0.7)(9.12) \text{ cfs} = \underline{\underline{6.38 \text{ cfs}}}$$

8. Equations for Variable Manning Roughness Coefficient

Although the “Flow in Partially Full Pipes” graph can be used to determine average velocity and flow rate for partially full pipe flow, as shown in Example #3, it would often be convenient to be able to make such calculations with an Excel spreadsheet. In order to do that, the following set of equations have been developed for n/n_{full} as a function of y/D , over the range from $0 \leq y/D \leq 1$:

$$0 \leq y/D \leq 0.03: \quad n/n_{\text{full}} = 1 + (y/D)/(0.3) \quad (3)$$

$$0.03 \leq y/D \leq 0.1: \quad n/n_{\text{full}} = 1.1 + (y/D - 0.03)(12/7) \quad (4)$$

$$0.1 \leq y/D \leq 0.2: \quad n/n_{\text{full}} = 1.22 + (y/D - 0.1)(0.6) \quad (5)$$

$$0.2 \leq y/D \leq 0.3: \quad n/n_{\text{full}} = 1.29 \quad (6)$$

$$0.3 \leq y/D \leq 0.5: \quad n/n_{\text{full}} = 1.29 - (y/D - 0.3)(0.2) \quad (7)$$

$$0.5 \leq y/D \leq 1: \quad n/n_{\text{full}} = 1.25 - (y/D - 0.5)(0.5) \quad (8)$$

Note that the first 5 equations are for $y/D \leq 0.5$ or less than half full. The last equation covers the entire range for more than half full pipe flow.

Example #5: Water is flowing through a 12" diameter corrugated metal pipe at a depth of 4 inches. The Manning roughness coefficient for full pipe flow in the corrugated metal pipe is: $n_{\text{full}} = 0.022$. Calculate the Manning roughness coefficient for the 4" deep flow in this pipe.

Solution: The given parameters are depth of flow: $y = 4$ inches and pipe diameter: $D = 12$ inches. Thus $y/D = 4/12 = 0.3333$. Since y/D is between 0.3 and 0.5, the equation for n/n_{full} is: $n/n_{\text{full}} = 1.29 - (y/D - 0.3)(0.2)$, as shown above.

$$n = n_{\text{full}}[1.29 - (y/D - 0.3)(0.2)] = (0.022)[1.29 - (0.3333 - 0.3)(0.2)]$$

$$\underline{\underline{n = 0.028}}$$

The calculation of n for given values of D , y , and n_{full} , is built into the course spreadsheet for both the “less than half full” and “more than half full” cases. The

section of the “less than half full” worksheet that includes calculation of n is shown in the screenshot below. If the diameter, D ; depth of flow, y ; and full pipe flow value for the Manning roughness coefficient, n_{full} , are entered, the spreadsheet calculates the value of n for that depth of flow. The screenshot shows the solution to Example #5, giving the same result: $n = 0.028$.

Pipe Diameter, D =	12	in	
Depth of flow, y =	0.33333	ft	
(must have $y \leq D/2$)			
Manning			
roughness, n_{full} =	0.022		
Channel bottom			
slope, S =	0.0003	ft/ft	
y/D =	0.333		
n/n_{full} =	1.28		
n =	0.028		

9. Flow Rate Calculation for Less than Half Full Flow

The cross-sectional area, **A**; wetted perimeter, **P**; and hydraulic radius, **R_h**, can be calculated for known pipe diameter and depth of flow using the equations that were

presented and discussed in Section 5. The appropriate equation from those presented in the previous section can be used to calculate the Manning roughness coefficient, n , for given n_{full} , y , and D . These values together with the pipe slope, S , can be used in the Manning equation to calculate the flow rate and velocity, as illustrated in the following example.

Example #6: Calculate the flow rate and average velocity for the 4" deep flow in the 12" diameter corrugated metal pipe from Example #5, if the pipe slope is 0.0085.

Solution: From the equations in section 5:

$$r = D/2 = 12/2 \text{ inches} = 6 \text{ inches} = 0.5 \text{ ft}$$

$$h = y = 4 \text{ inches} = 0.3333 \text{ ft}$$

$$\theta = 2 \arccos [(r - h)/r] = 2 \arccos [(0.5 - 0.3333)/0.5] = 2.462 \text{ radians}$$

$$A = r^2(\theta - \sin\theta)/2 = (0.5^2)[2.462 - \sin(2.462)]/2 = 0.2292 \text{ ft}^2$$

$$P = r\theta = (0.5)(2.462) = 1.231 \text{ ft}$$

$$R_h = A/P = 0.2292/1.231 = 0.1862 \text{ ft}$$

From Example #4: $n = 0.028$

Now the Manning equation can be used to calculate Q :

$$Q = (1.49/n)A(R_h^{2/3})S^{1/2} = (1.49/0.028)(0.2292)(0.1862^{2/3})(0.0085^{1/2})$$

$$Q = \underline{\underline{0.364 \text{ cfs}}}$$

$$V = Q/A = 0.367/0.2292 = \underline{\underline{1.59 \text{ ft/sec}}}$$

As expected, this problem can be solved using the course spreadsheet. A portion of the “Q_less than half full” worksheet is shown below with the solution to Example #6, resulting in the same values for Q and V.

<u>Inputs</u>			<u>Calculations</u>		
Pipe Diameter, D =	12	in	Pipe Diameter, D =	1	ft
Depth of flow, y =	0.33333	ft	Pipe Radius, r =	0.5	ft
(must have $y \leq D/2$)					
			Circ. Segment Height, h =	0.333333	ft
Manning					
roughness, n_{full} =	0.022		Central Angle, θ =	2.46	radians
Channel bottom					
slope, S =	0.0085	ft/ft	Cross-Sect. Area, A =	0.23	ft ²
			Wetted Perimeter, P =	1.2	ft
y/D =	0.333		Hydraulic Radius, R =	0.19	ft
n/n_{full} =	1.28		Discharge, Q =	0.364	cfs
n =	0.028		Ave. Velocity, V =	1.59	ft/sec

Example #7: Calculate the flow rate and average velocity for water flow 20 mm deep in a 100 mm diameter corrugated metal pipe ($n_{full} = 0.022$), if the pipe slope is 0.0085.

Solution: From the equations in section 5:

$$r = D/2 = 100/2 \text{ mm} = 50 \text{ mm} = 0.05 \text{ m}$$

$$h = y = 20 \text{ mm} = 0.02 \text{ m}$$

$$\theta = 2 \arccos [(r - h)/r] = 2 \arccos [(0.05 - 0.02)/0.05] = 1.854 \text{ radians}$$

$$A = r^2(\theta - \sin\theta)/2 = (0.05^2)[1.854 - \sin(1.854)]/2 = 0.001117 \text{ m}^2$$

$$P = r\theta = (0.05)(1.854) = 0.0927 \text{ m}$$

$$R_h = A/P = 0.001117/0.0927 = 0.01205 \text{ m}$$

For $y/D = 20/100 = 0.2$, from Eqn (5): $n/n_{\text{full}} = 1.22 + (0.2 - 0.1)(0.6) = 0.028$

Now the Manning equation can be used to calculate Q :

$$Q = (1.49/n)A(R_h^{2/3})S^{1/2} = (1.00/0.028)(0.001117)(0.01205^{2/3})(0.0085^{1/2})$$

$$Q = \underline{\underline{0.000193 \text{ m}^3/\text{s}}}$$

$$V = Q/A = 0.000193/0.001117 = \underline{\underline{0.173 \text{ m/sec}}}$$

10. Flow Rate Calculation for More than Half Full Flow

The cross-sectional area, A ; wetted perimeter, P ; and hydraulic radius, R_h , can be calculated for known pipe diameter and depth of flow using the equations that were presented and discussed in Section 6. The appropriate equation from those presented in the previous section can be used to calculate the Manning roughness coefficient, n , for given n_{full} , y , and D . These values together with the pipe slope, S , can be used in the Manning equation to calculate the flow rate and velocity, as illustrated in the following example.

Example #8: Calculate the flow rate and average velocity for 10" deep flow in the 12" diameter corrugated metal pipe from Example #4, if the pipe slope is 0.0085.

Solution: From the equations in section 6:

$$r = D/2 = 12/2 \text{ inches} = 6 \text{ inches} = 0.5 \text{ ft}$$

$$h = 2r - y = (2)(0.5) - 10/12 = 0.1667 \text{ ft}$$

$$\theta = 2 \arccos [(r - h)/r] = 2 \arccos [(0.5 - 0.1667)/0.5] = 1.682 \text{ radians}$$

$$A = \pi r^2 - r^2(\theta - \sin\theta)/2 = \pi 0.5^2 - (0.5^2)[1.682 - \sin(1.682)]/2 = 0.6994 \text{ ft}^2$$

$$P = 2\pi r - r\theta = 2*\pi()*0.5 - (0.5)(1.682) = 2.300 \text{ ft}$$

$$R_h = A/P = 0.6994/2.300 = 0.3040 \text{ ft}$$

From Example #4: $n = 0.028$

Now the Manning equation can be used to calculate Q :

$$Q = (1.49/n)A(R_h^{2/3})S^{1/2} = (1.49/0.028)(0.6994)(0.3040^{2/3})(0.0085^{1/2})$$

$$\underline{Q = 1.55 \text{ cfs}}$$

$$V = Q/A = 1.55/0.6994 = \underline{2.22 \text{ ft/sec}}$$

11. Review of Normal Depth Calculation

For a constant flow rate through a channel with constant bottom slope, cross-sectional shape and size, and Manning roughness coefficient, the depth of flow will be constant at a depth called the normal depth. The procedure for determining the normal depth is the same for gravity flow through partially full pipes as it is for

open channel flow with cross-sectional shapes like rectangular or trapezoidal. The normal depth can be determined by rearranging the Manning equation to:

$$A(R_h^{2/3}) = Qn/1.49(S^{1/2}) \quad (9)$$

For flow in a channel with specified **Q**, **n**, and **S**, the right hand side of the equation has a constant value that can be calculated. The left hand side of the equation can be written as a function of depth of flow, **y**, for specified channel shape and size. A iterative solution is typically required to find the value of **y** that makes the two sides of the equation equal. This type of calculation is illustrated in Example #9 below.

Example #9: Determine the normal depth for water flowing at a rate of 18 cfs in a rectangular channel that has a bottom slope of 0.00084, bottom width of 4 ft, and Manning roughness coefficient of 0.013.

Solution: For the given rectangular channel: $A = by_o = 4y_o$,
 $P = b + 2y_o = 4 + 2y_o$, and $R_h = A/P = 4y_o/(4 + 2y_o)$

Substituting values and expressions into Equation (9), the rearranged Manning equation gives:

$$(4y_o)[4y_o/(4 + 2y_o)]^{2/3} = (18*0.013)/(1.49*0.00084^{1/2}) = 5.419$$

This equation requires an iterative (trial and error) solution, because it can't be solved explicitly for y_o . The table below was printed from an Excel spreadsheet that was used to carry out the iterative solution. The steps leading to the conclusion that $y_o = 1.5$ ft (accurate to 2 significant figures) are shown in the table. The table shows that $y_o = 1.50$ ft gives a value for $(4y_o)[4y_o/(4 + 2y_o)]^{2/3}$ that is closer to the target value of 5.419 than that given by either $y_o = 1.49$ or $y_o = 1.51$. Thus:

$$\underline{y_o = 1.50 \text{ ft} = 18 \text{ in}}$$

		Difference from
y_o , ft	$A \cdot R^{2/3}$	Target Value
1	3.053	-2.366
2	8.000	2.581
1.50	5.414	-0.005
1.49	5.364	-0.054
1.51	5.464	0.045

12. Normal Depth for Less than Half Full Flow

Calculation of normal depth for partially full pipe flow is slightly more complicated than its calculation for a rectangular or trapezoidal channel, because for the partially full pipe flow, the Manning roughness coefficient isn't a constant, but rather depends upon the depth of flow, y . The Manning equation can thus be rearranged to:

$$A(R_h^{2/3})/n = Q/1.49(S^{1/2}) \quad (10)$$

The right hand side of this equation is constant and the left hand side is a function of the normal depth, y_o , so iterative solution of this equation can be used instead of equation (9) to determine the normal depth for partially full pipe flow.

Example #10: Find the depth of flow (normal depth) for 0.2 cfs of water flowing through a 12 inch diameter concrete pipe ($n_{full} = 0.013$) with a pipe slope of 0.0085.

Solution: The given parameters are **D** = 12 inches, **Q** = 0.2 cfs, **S** = 0.0085, and **n_{full}** = 0.013. The right hand side of equation (10) is:

$$Q/1.49(S^{1/2}) = 0.2/(1.49*0.0085^{1/2}) = 1.456$$

The equations needed to calculate $A(R_h^{2/3})/n$ for a given value of y_o are:

r = **D/2**, **h** = **y**, **θ** = **2 arccos[(r – h)/r]**, **A** = **r²(θ – sinθ)/2**, **P** = **rθ**, and the set of equations for **n/n_{full}** from section 8.

The iterative solution can be done by hand to find the value of y_o that makes $A(R_h^{2/3})/n$ equal to 1.456 to the degree of accuracy desired. However, the use of a spreadsheet makes the repetitive calculations much easier. The table below is copied from The “Normal Depth_less than half” worksheet in the course spreadsheet. It shows the solution:

$$\underline{\underline{y_o = 0.188 \text{ ft} = 2.25 \text{ in}}}$$

Inputs				Calculations			
Pipe Diameter, D =	12	in		Pipe Diameter, D =	1	ft	
Manning roughness, n_{full} =	0.013			Pipe radius, r =	0.5	ft	
Channel bottom slope, S =	0.0085	ft/ft		The Manning equation can be rearranged to:			
Volumetric Flow Rate, Q =	0.2	cfs		$Q/(1.49 \cdot S^{1/2}) = (A \cdot R^{2/3})/n$			
Iterative (trial & error) Solution:				$Q/(1.49 \cdot S^{1/2}) =$	1.456	= target value	
(Select values of y_o , to find the value of y_o that makes						for $(A \cdot R^{2/3})/n$	
$(A \cdot R^{2/3})/n$ as close to the target value as possible)							
							difference from
y_o , ft	y_o/D	θ , radians	A, ft ²	n	P, ft	$(A \cdot R^{2/3})/n$	target value
0.5	0.500	3.142	0.39	0.0163	1.57	9.590	8.134
0.2	0.200	1.855	0.11	0.0168	0.93	1.628	0.172
0.19	0.190	1.804	0.10	0.0166	0.90	1.485	0.029
0.189	0.189	1.799	0.10	0.0166	0.90	1.470	0.014
0.188	0.188	1.794	0.10	0.0165	0.90	1.455	-0.001
0.187	0.187	1.789	0.10	0.0165	0.89	1.440	-0.016
NOTE: For Q = 0.2 cfs, this set of calculations shows that $y_o = 0.188$ ft							
(accurate to 3 significant figures)							

NOTE: For this example, the normal depth turned out to be less than half of the pipe diameter, so the pipe was indeed flowing less than half full. If the flow rate were increased above the 0.2 cfs value used in this example, then the normal depth would be greater than 0.188 ft. At some value of flow rate the normal depth would be one half of the pipe diameter. That is the maximum normal depth that can be determined with the “Normal Depth_less than half” worksheet. If you are trying to

find a normal depth with this worksheet and find that you must use values of y_o greater than half the pipe diameter to cause the “difference from target value” to decrease, then you need to switch to the “Normal Depth_more than half” worksheet to find the normal depth. The equations for calculating A and P are different for “more than half full” pipe flow as discussed in the next section, so the “Normal Depth_less than half full” will not give the correct value for normal depth in cases where the normal depth is greater than half of the pipe diameter.

13. Normal Depth for More than Half Full Flow

Determination of normal depth for more than half full pipe flow is nearly the same as that just described for less than half full pipe flow. The equations for calculating A, P, and R_h are a bit different for the case where the pipe is flowing more than half full, as given in Section 6. Calculation of n/n_{full} is simpler, because the single equation, $n/n_{full} = 1.25 - (y/D - 0.5)(0.5)$, applies over the entire range $0.5 < y/D < 1.0$.

Example #11: Find the depth of flow (normal depth) for 2.5 cfs of water flowing through a 12 inch diameter concrete pipe ($n_{full} = 0.013$) with a pipe slope of 0.0085.

Solution: The given parameters are $D = 12$ inches, $Q = 2.5$ cfs, $S = 0.0085$, and $n_{full} = 0.013$. The right hand side of equation (10) is:

$$Q/1.49(S^{1/2}) = 2.5/(1.49*0.0085^{1/2}) = 18.20$$

The equations needed to calculate $A(R_h^{2/3})/n$ for a given value of y_o are:

$$r = D/2, \quad h = 2r - y, \quad \theta = 2 \arccos[(r - h)/r], \quad A = \pi r^2 - r^2(\theta - \sin\theta)/2,$$

$$P = 2\pi r - r\theta, \quad \text{and} \quad n/n_{full} = 1.25 - (y/D - 0.5)(0.5)$$

The diagram on the next page is from the "Normal Depth_more than half" worksheet in the course spreadsheet. It shows the calculation of $A(R_h^{2/3})/n$ for selected values of y_o , with the flow configuration in this example, in order to find the normal depth. As shown in the spreadsheet screenshot, the resulting normal depth is:

$$\underline{y_o = 0.719 \text{ ft} = 8.63 \text{ in}}$$

NOTE: For this example, the normal depth of 0.719 ft is indeed more than half of the pipe diameter, so the use of the "Normal Depth_more than half full" worksheet was appropriate. If you start the process of determining normal depth with the "Normal Depth_more than half full" worksheet and find that you need to use values of y_o that are less than half of the pipe diameter to cause the "difference from target value" to decrease, then you should switch to the "Normal Depth_less than half" worksheet.

Partially Full Pipe Flow Calculations - U.S. Units							
IV. Calculation of Normal Depth for Pipes More Than Half Full							
Instructions: Enter values in blue boxes. Spreadsheet calculates values in yellow boxes							
Inputs				Calculations			
Pipe Diameter, D =	12	in		Pipe Diameter, D =	1	ft	
Manning roughness, n_{full} =	0.013			Pipe radius, r =	0.5	ft	
Channel bottom slope, S =	0.0085	ft/ft		The Manning equation can be rearranged to:			
Volumetric Flow Rate, Q =	2.5	cfs		$Q/(1.49 \cdot S^{1/2}) = (A \cdot R^{2/3})/n$			
Iterative (trial & error) Solution:				$Q/(1.49 \cdot S^{1/2}) =$ 18.20 = target value			
(Select values of y_o , to find the value of y_o that makes $(A \cdot R^{2/3})/n$ as close to the target value as possible)				for $(A \cdot R^{2/3})/n$			
y_o , ft	h, ft	θ , radians	A, ft ²	n	P, ft	$(A \cdot R^{2/3})/n$	difference from target value
0.8	0.20	1.855	0.67	0.0143	2.21	21.3	3.106
0.7	0.30	2.319	0.59	0.0150	1.98	17.5	-0.744
0.71	0.29	2.275	0.60	0.0149	2.00	17.9	-0.343
0.72	0.28	2.230	0.61	0.0148	2.03	18.3	0.056
0.719	0.28	2.235	0.60	0.0148	2.02	18.2	0.016
0.718	0.28	2.239	0.60	0.0148	2.02	18.2	-0.023
NOTE: For Q = 2.5 cfs, this set of calculations shows that $y_o = 0.719$ ft							
(accurate to 3 significant figures)							

14. Summary

Calculation of flow rate and average velocity, or determination of normal depth at a given flow rate, for partially full pipe flow can be carried out with the Manning equation in a manner similar to such calculations for traditional open channel cross-sections, like rectangular or trapezoidal. Calculations for partially full pipe flow are complicated by two factors: i) the equations for calculating A , P and R_h are somewhat more complicated and ii) the Manning roughness coefficient must be considered to vary as a function of the ratio of depth of flow to diameter (y/D) in order to make accurate calculations. The equations needed and example calculations for partially full pipe flow are presented and discussed in this course.

15. References

1. Bengtson, Harlan H., an online article, "[Partially Full Pipe Flow Calculations with Excel Spreadsheets.](#)"
2. Camp, T.R., "Design of Sewers to Facilitate Flow," *Sewage Works Journal*, 18 (3), 1946
3. Chow, V. T., *Open Channel Hydraulics*, New York: McGraw-Hill, 1959.
4. Steel, E.W. & McGhee, T.J., *Water Supply and Sewerage*, 5th Ed., New York, McGraw-Hill Book Company, 1979
5. ASCE, 1969. *Design and Construction of Sanitary and Storm Sewers*, NY