Time Designation, GPS, and Triangulation Surveys

PDH Credits:

4 PDH

Course No.:

GPS101

Publication Source:

US Navy
Engineering Aid Basics

"Chapter 14 – Time Designation, GPS, and Triangulation"

Pub. # NAVEDTRA 14336A

Chapter 14

Time Designation, GPS, and Triangulation

Topics

1.0.0 Time

2.0.0 Satellite Surveying Systems

3.0.0 Triangulation

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Overview

This chapter discusses important aspects of your duties as a Triangulation Survey Party Chief. Your understanding of time designations and triangulation methods is essential in leading your party to accurate survey results. We will explain the use of the different designations of time, such as solar time, zone time, and Greenwich Mean Time.

In the discussion of triangulation, we will explain the purpose and kinds of triangulation networks, the steps involved in a triangulation survey, and the computations involved in establishing horizontal control points using triangulation.

This chapter also addresses satellite surveying systems and how they are used in locating point positions on the surface of the earth though observation of data received from Global Positioning Satellites (GPS).

Objectives

When you have completed this chapter, you will be able to do the following:

1. Describe the methods of measuring and recording time.
2. Describe the use of GPS systems in surveying.
3. Describe the application of triangulation in surveying.

Prerequisites

None

1.0.0 TIME

There are two naturally occurring earth cycles on which time measurements are based, the year and the day. The year is defined as the time required for Earth to complete one revolution around the Sun, while the day is the time required for Earth to complete one turn upon its axis. Our Earth needs 365 days plus approximately 6 hours to go around the Sun once. This fractional day of 6 hours is accommodated on the calendar systems by the addition of an extra day every fourth year, called leap year. The addition of the leap year keeps our calendars synchronized with the seasons.

Because Earth, while turning upon its axis, also moves around the Sun, there are different ways of determining a day's duration. For our purposes of time measurement a day is defined as the time interval between the moments when the sun reaches the same point in the sky, directly overhead.

1.1.0 Solar Time

The sun is the most commonly used reference point for reckoning time, and time reckoned by the sun is solar time. Time reckoned according to the position of the actual physical sun is solar apparent time. When the sun is directly over a meridian, it is noontime, local apparent time, along that meridian. At the same instant, it is midnight, local apparent time, on the meridian 180° away from that meridian, on the opposite side of the earth.

The time required for a complete revolution of the earth on its axis is a constant 24 hours with regard to a particular point on the earth; however, this time varies slightly with regard to the point's position with relation to the actual sun. Therefore, days reckoned by apparent time (that is, the position of the actual sun) vary slightly in length. This difficulty can be avoided by reckoning time according to a mean position of the sun, and this is called mean time. By mean time the interval from noon to noon along any meridian is always the same 24 hours.

But if all clocks were actually set by mean solar time, there would be time differences between the closely located geographic locations within the same meridians that would be "correct" but a major nuisance. For example, a clock in Las Vegas, NV, correctly showing mean solar time for its location (this would be ***local civil time***), would be slightly ahead of a clock in China Lake, CA. The China Lake clock would be slightly ahead of a clock in Bakersfield, CA, which in turn would be ahead of a clock in San Francisco. This condition prevailed until 1884, when a system of standard time was adopted by the International Meridian Conference. Earth's surface was divided into 24 zones.

1.1.1 Zone Time

Under the zone time system, the earth has been divided along meridians into 24 time zones. The starting point is the Greenwich, England meridian, lying at 0° longitude. Every meridian east or west of Greenwich that is numbered 15° or a multiple of 15° (such as 30° east or west, 45° east or west, 60° east or west, and so on) is designated as a ***standard time meridian*****.** Each time a meridian runs through the center of its time zone, which means that the zone extends for 7°30' on each side of the meridian. In each zone, the time is the same throughout the zone (*Figure 14-1*).

There is a 1 hour difference in time between a particular zone and the adjacent zone. When determining time in different zones, it is helpful to remember this phrase: time is later as you move eastward. So, if it is 1200 in your zone, it is 1300 in the next zone to the east and 1100 in the next zone to the west.

1.1.1.1 Greenwich Mean Time

The time listed in most of the computational tables used in celestial observations is Greenwich Mean Time (GMT) — meaning the zone time in the Greenwich standard time zone. You must know how to convert the zone time at which you made a particular observation to Greenwich Mean Time. The procedure is as follows.

Each of the time zones has a number that is called the zone description (ZD). The Greenwich zone is numbered 0. The others are numbered from 1 through 12, east or west of Greenwich. To determine the ZD for any point on the earth, you divide the longitude by 15.

If the remainder is greater than 7° 30', the quotient plus 1 is the ZD. Suppose, for example, that the longitude at the point of your observation is 142°41′W. Divide this longitude value by 15 and you get 9, with a remainder of 7°41'. Since the remainder is greater than 7°30', the ZD is 9 + 1, or 10.

Zones east of Greenwich are minus, and zones west of Greenwich are plus. To convert the zone time of an observation to the corresponding Greenwich meantime, you apply the ZD according to its sign to the zone time. For example, suppose the longitude at your point of observation is 75°15' 37"E and the zone time is $16^h\ 23^m\ 14^s$. Divide the longitude by 15 and you get 5, with less than 7°30' left over, the longitude is east; therefore, the ZD is -5, and the GMT of the observation is $16^h23^m14^s$ -5^h, or $11^h23^m14^s$. Suppose now that the longitude of the point of observation is 68° 19′ 22″ W and the zone time of the observation is $10^h\ 15^m\ 08^s$. Divide the longitude by 15 and you get 4, with more than 7°30" left over. The ZD is therefore +5 and the GMT of the observation is $10^h15^m08^s + 5^h$, or $15^h15^m08^s$.

1.1.1.2 Zone Time and Date

It may be the case that the date at Greenwich and the date at the point of observation are not the same at the time of observation. Suppose that on 1 May you are in longitude 176°15′22″W and the zone time of your observation is 16h 24m 1s. The ZD is +12. GMT of the observation is therefore 16h 24m 11s + 12 or 28h 24m 11s. However, 28h 24m 11s on 1 May means 04h 24m 11s on 2 May, and you would refer to the tables for that GMT and date.

Suppose now that on 1 May you are in longitude 47°32′55″E and the zone time of the observation is 02h 15m 27s, but 02h 15m 27s on 1 May can be considered as 26h 15m 27s on 30 April. Therefore, GMT for the observation was 25h 15m 27s – 3h, or 23h 15m 27s, on 30 April.

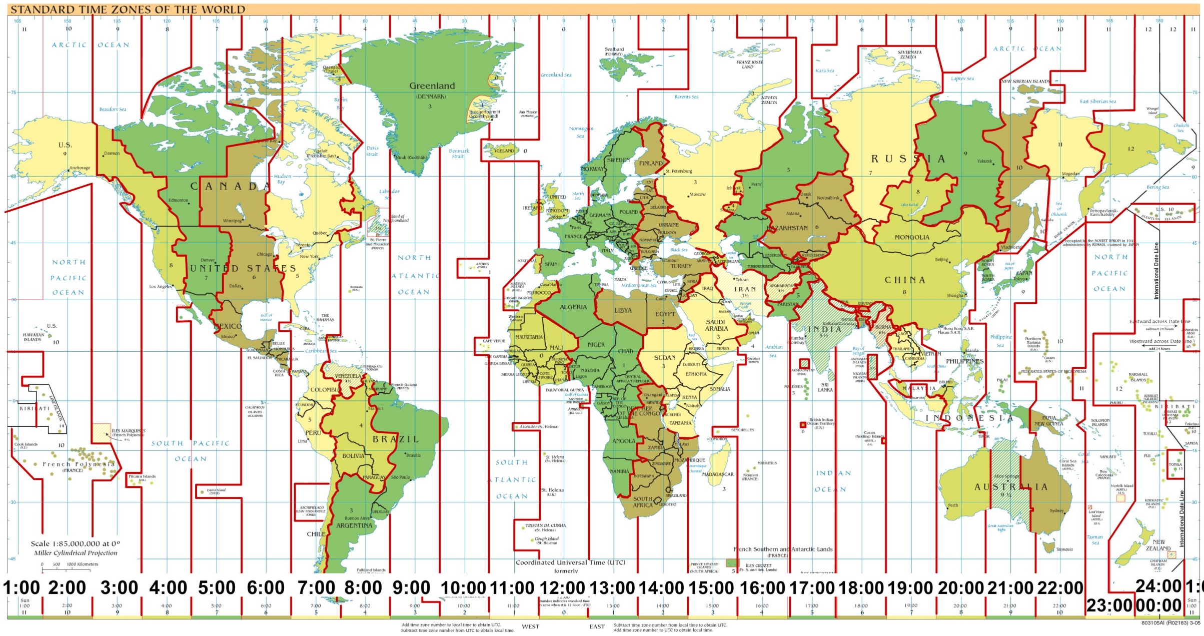


Figure 14-1 — 24 time zones.

1.1.1.3 Importance of Exact Time

The importance of recording the exact time at which an observation is made may be illustrated as follows. Suppose a ship's navigator makes an error of only 1 minute in his or her time. This could produce an error of as much as 15 miles in the location of the computed and plotted line of position. A 1-minute time error produces a 15-minute error in longitude regardless of the latitude, and on the equator, a minute of longitude equals a nautical mile.

You must time the observation to the nearest second, and for this purpose, you must have an accurate watch. It is best that you have an accurate ordinary watch plus a stopwatch. You should set the ordinary watch to exact time shortly before the time of observation. Correct standard time can be obtained from a clock known to be closely regulated, or preferably from time signals broadcast by the U.S. Naval Observatory.

Remember, in localities under daylight savings time, the time is 1 hour faster than standard time.

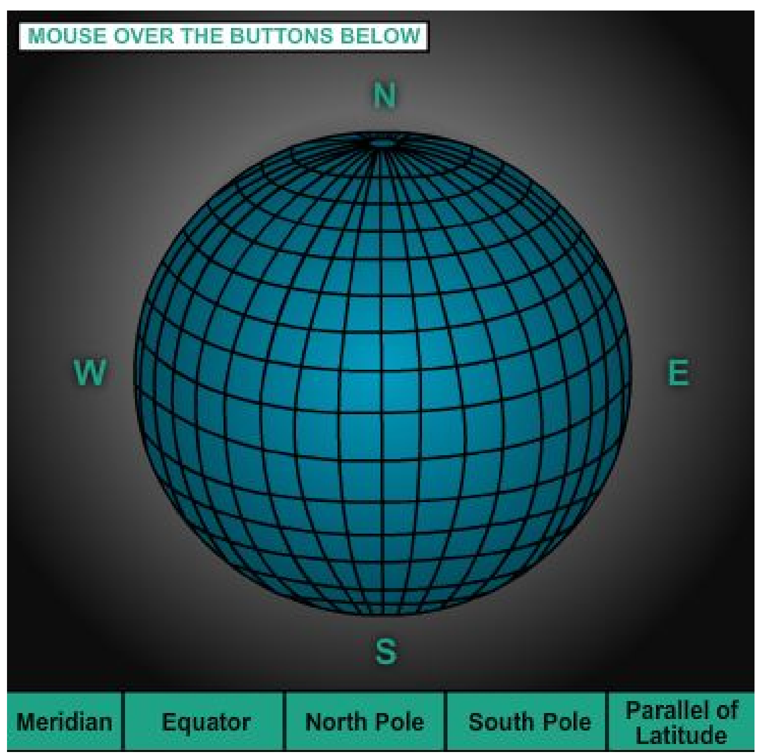


Figure 14-2 — Reference lines.

1.1.1.4 Terrestrial System

Although the earth is not actually a true sphere, it is presumed to be such for the purpose of astronomy. Astronomic determinations are based on the relationships that exist among sets of spherical coordinates. The terrestrial system is stated in latitude and longitude. The terrestrial system of coordinates refers to the location of points on the terrestrial sphere (the earth). In the terrestrial system, the fundamental reference lines are the axis of the earth's rotation and the earth's equator (*Figure 14-2*). The ends of the axis of rotation are known as the poles, designated as the North and South. A great circle passing through both poles is called a meridian. The equator is a great circle about the earth equidistant from the poles and perpendicular to the axis of rotation. Through any point removed from the equator, a circle whose plane is parallel to that of the equator is called a parallel of latitude. The numerical value of the parallels defines latitude and that of the meridians defines longitude.

As shown in *Figure 14-3*, geographic latitude of a point may be defined as its angular distance above or below the equator. Latitudes are expressed in degrees and are measured from 0° to 90° north or south. The conventional symbol for latitude used in computation is Greek letter Φ (phi).

As shown also in *Figure 14-3*, the longitude of a point is the angular distance measured along the equator between the meridian passing through a point and a reference meridian. The chosen reference meridian is the Greenwich meridian that passes through Greenwich, England. That meridian is known as the primary or prime meridian. Longitude is also expressed in degrees but is measured from 0° to 180° west or east from the prime meridian. The conventional symbol for longitude is the Greek letter Λ (lambda).

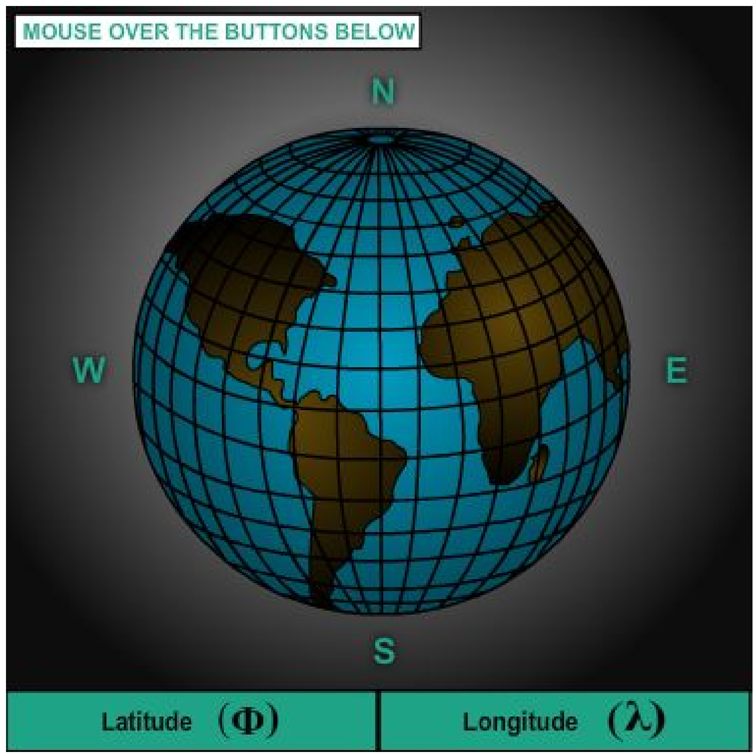


Figure 14-3 — Latitude, longitude, and reference lines.

Test your Knowledge (Select the Correct Response)

1. What are the two naturally occurring earth cycles on which time measurements are based?

   A. Day and month
   B. Year and day
   C. Hour and minute
   D. Hour and month

2. **(True or False)** The sun is the most commonly used reference point for reckoning time.

   A. True
   B. False

3. Under the zone time system, the earth has been divided along meridians into how many time zones?

   A. 8
   B. 12
   C. 16
   D. 24

2.0.0 SATELLITE SURVEYING SYSTEMS

In this section we will discuss satellite surveying systems, which are an offshoot of the space program and the U.S. Navy’s activities related to navigation. Since their development, satellite surveying systems have been successfully used in nearly all areas of surveying and are capable of producing extremely accurate results.

The first generations of satellite surveying systems were the Doppler positioning systems. The success of the Doppler systems led to the U.S. Department of Defense development of a new navigation and positioning system using NAVSTAR (Navigation Satellite Timing and Ranging) satellites. This development ushered in the second generation of satellite surveying systems known as the Global Positioning System (GPS).

2.1.0 Global Positioning Systems

Because of its superiority, the global positioning system has replaced the use of the Doppler positioning system; however, like the Doppler system, the global positioning system is based on observations of satellites.

GPS satellites are in near-circular orbits around the globe at an altitude of approximately 12,400 miles above the earth (*Figure 14-4*). These satellites transmit unique signals that are encoded with information that enables ground receivers to measure the travel time of the signals from satellite to receiver. That travel time is then converted to distance using the velocity of electromagnetic energy through the atmosphere.

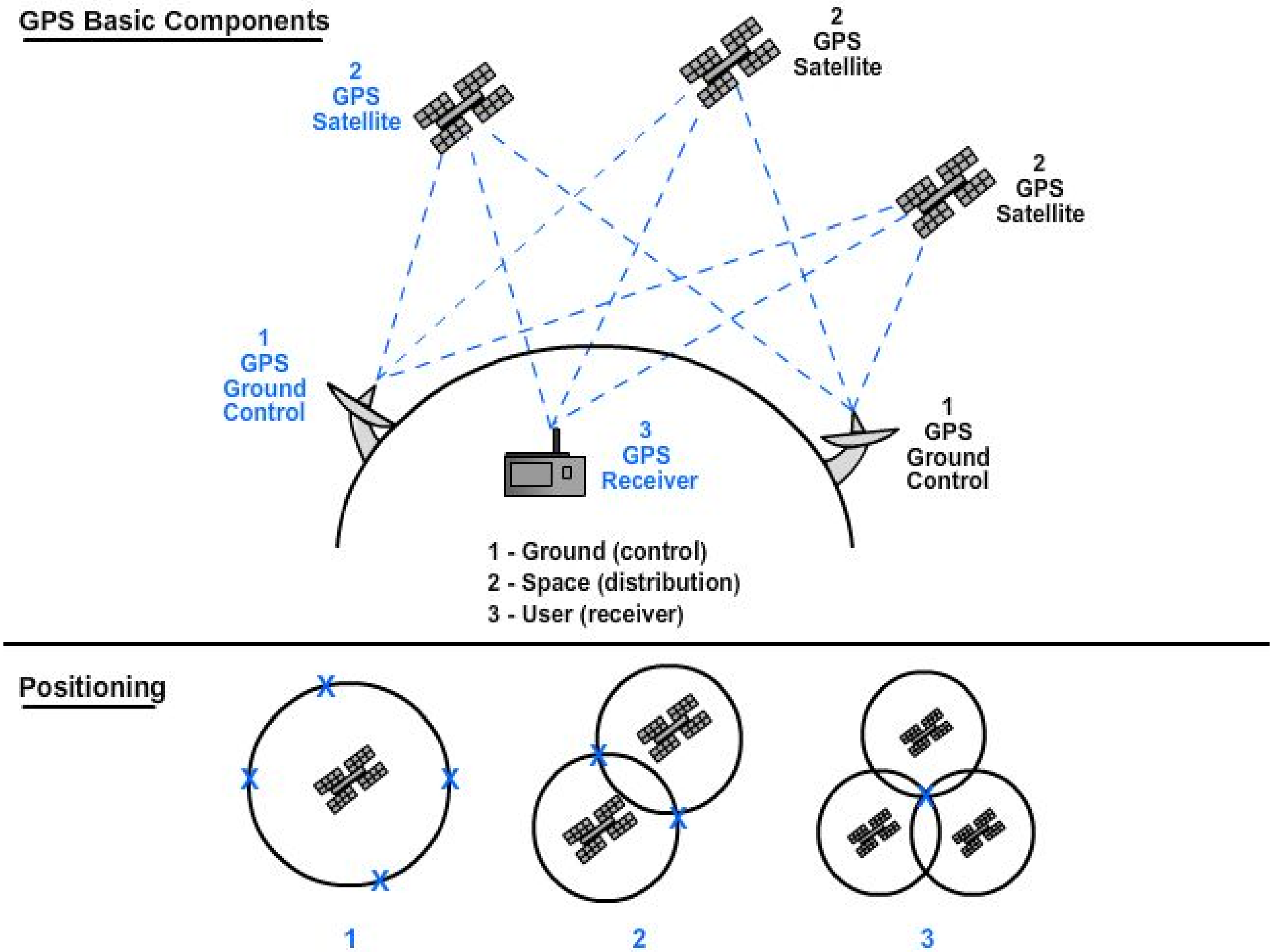


Figure 14-4 – GPS basic components.

Determining point locations using GPS procedures essentially consists of measuring distances from points at unknown locations to satellites whose positions are known at the instant of observation. In concept this is identical to performing resection using distances that are measured from a point of unknown location to three or more stations whose positions are known.

Global Navigation Satellite Systems (GNSS) refers to the collective worldwide civil positioning, navigation, and timing determination capabilities available from one or more satellite constellations. The world's GNSS include the United States' GPS, the Russian Federation's GLONASS and the European Union's Galileo, which are all undergoing modernization.

2.2.0 GPS Fundamentals

GPS is funded by and controlled by the U. S. Department of Defense (DOD). While there are many thousands of civil users of GPS world-wide, the system was designed for and is operated by the U. S. military. GPS provides specially coded satellite signals that can be processed in a GPS receiver, enabling the receiver to compute position, velocity, and time. Four GPS satellite signals are used to compute positions in three dimensions and the time offset in the receiver clock.

It is a one-way (listen only) system, in which the satellites transmit signals but are unaware who is using the signal (no receiving function). The user (or listener) does not transmit a signal, and therefore cannot be detected by the enemy (military context), and cannot be charged for using the system (civilian context).

As GPS is a multi-satellite system, there are always a number of satellites visible simultaneously anywhere on the globe and at any time.

GPS can support a number of positioning and measurement modes in order to satisfy simultaneously a variety of users, from those requiring only navigation (decameter) accuracies to those demanding very high (millimeter - centimeter) accuracies.

2.2.1 GPS Today

The evolution of surveying from chains, bars, tapes, theodolites, and levels through the EDMI of the 1950s to the GPS antennae and receivers of today has produced a dramatic increase in the speed and accuracy with which positioning can be accomplished.

GPS was rapidly adapted for surveying, as it can give a position (Latitude, Longitude and Height) directly, without the need to measure angles and distances between intermediate points. Survey control can now be established almost anywhere as it is only necessary to have a clear view of the sky so the signal from the GPS satellites can be received clearly.

Today’s GPS satellites transmit two carrier frequencies that are commonly referred to as L1 and L2. Both of which contain codes that provide positioning, timing, and navigation information. Utilizing these frequencies and codes allows GPS receivers to track several satellite signals at the same time, so that precise positioning can be calculated anywhere on earth.

Table 14-1 ─ GPS Carriers

Carrier	Frequency	Code
L1	1575.42 MH_Z	C/A and P/Y
L2	1227.6 MH_Z	L2C and P/Y
L5	1176.45 MH_Z	L5 Civil

As shown in *Table 14-1*, the L1 carrier contains Coarse/Acquisition (CA) code, which is commercially available. The L2 carrier contains only the P/Y, which is an encrypted code reserved for military use.

Initially, commercial GPS receivers could receive only the civilian L1 carrier, and to achieve survey-accuracy positioning, surveyors had to perform additional error allowances that restricted accuracy. Today, by utilizing GPS receivers that can utilize both of the L1 and L2 carriers that also contain the newest “civil signal” on the L2 carrier, called L2C, centimeter-level accuracy is achievable for RTK work. The first L2C capable satellite was launched in 2005, but as of 2007 a new L5 carrier is available.

2.3.0 Advantages of GPS Over Conventional Surveying Methods

There are several advantages of the GPS satellite surveying techniques:

- Intervisibility between stations is not necessary.
- Because GPS uses radio frequencies to transmit the signals, the system is independent of weather conditions.
- If the same field and data reduction procedures are used, position accuracy is largely a function of interstation distance, and not of network "shape" or "geometry.”
- Because of the generally homogeneous accuracy of GPS surveying, network planning in the classical sense is no longer relevant. The points are placed where they are required (for example, in a valley), and need not be located at evenly distributed sites atop mountains to satisfy intervisibility, or network geometry, criteria.
- Because of the two advantages of not requiring intervisibility of stations, or following a conventional network design strategy, GPS surveying is more efficient, more flexible and less time consuming a positioning technique than using terrestrial survey technologies.
- GPS can be used around-the-clock.
- GPS provides three-dimensional information.
- High accuracies can be achieved with relatively little effort, unlike conventional terrestrial techniques. The GPS instrumentation, and to some extent the data processing software, are similar whether accuracies at the 1 part in 10^4 or 1 part in 10^6 level are sought.

The very latest in GPS-based surveying involves combining both conventional survey instruments (optical) and GPS receivers. Survey data gathered in such a manner is also stored and managed via new electronic methods that are usually equipment vendor specific with respect to the programs and electronics used to store and process gathered data. Current Seabee systems are based on the equipment provided by the Trimble Geomatics and Engineering Company. The Navy has set up dedicated courses of instruction on the use of this equipment by Seabees in the performance of surveying and will be your best source for how to use this equipment in the field. What follows is an overview of what is commonly referred to as Integrated Surveying using the latest in conventional and GPS electronic surveying equipment.

GPS is similar in some ways to triangulation discussed later in this chapter, except that the known positions are now the GPS satellites in space. The equipment and calculations are extremely complex, but for the user the process is generally very simple, as the GPS receiver electronics and the software programs take care of the calculations. In the commonly available systems the Seabees are using, the GPS receiver almost instantly works out its position (Latitude, Longitude and Height) with an uncertainty of a few centimeters.

2.4.0 Kinematic GPS

There are many variations on this type of GPS surveying, but basically it is similar to the GPS baseline method, except that while one GPS receiver remains on a known position (Base Station), the other (rover) moves between points and needs to be at each point for only a few seconds. Corrections to the GPS data (based on the known Base Station position and its position computed from the GPS) may be immediately transmitted from the receiver on the Base Station to the receiver at the other end of the line (the rover station). The position of the rover station can then be computed and stored within a few seconds.

The most important tool for this type of surveying is the Total Station. It is given this name because it incorporates GPS, a distance meter for measuring distances, and a theodolite for measuring angles into one instrument. The total station measures by sending a beam of infrared light toward a prism target, usually supported either by a tripod or a pole with a GPS receiver on top. The light reflects off the prism directly back to the total station. By measuring the time it takes for the light to return, the total station calculates the distance away that the prism is.

The information that the total station measures (angles and distances) is recorded in a data collector for later downloading into a computer. The data collector also doubles as a field computer, enabling calculation of coordinate geometry in the field and determining the proper position of stakes. Elevation is derived with the total station by using the geometry of measured angles and distances, and by using the Global Positioning System by intersecting vectors from satellites in space.

2.4.1 Methods of GPS Total Station Surveys

Construction Site Stakeout usually involves a large number of markers that must be placed and many components that must be positioned. If control points exist (they can often be damaged or covered by equipment, material, vehicles, etc.), set up the total station there and obtain a GPS position fix. If no control point exists, set up the total station wherever it is convenient. Following set up at a first point, set up at a second point, fix the position, use the first point for orientation, and stake out from the second point. Work in this way, establishing pairs or groups of points from which to stake out (*Figure 14-5, View A*).

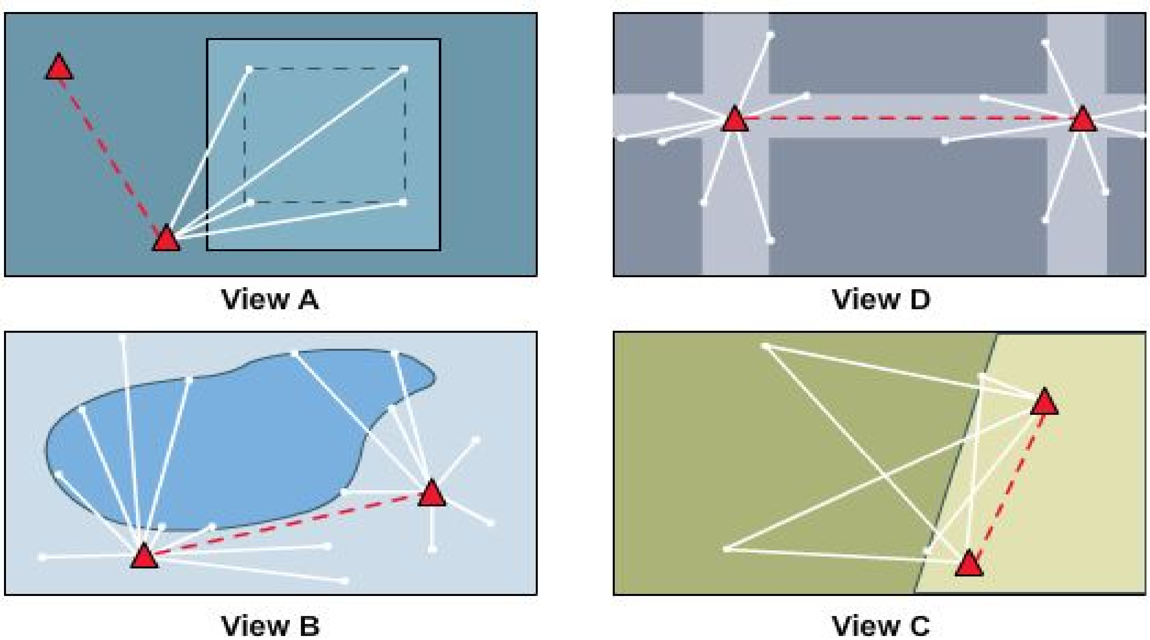


Figure 14-5 — Different types of GPS surveys.

Remote Area Topographic Surveys involve setting up the total station where it is convenient and determining the position with GPS. Orient to a second point that will be used but is not coordinated yet. Survey the topographical details from the first station. Set up at the second point and determine the position with RTK and GPS, as the bearing between the points is now known. The software program that ties the total station and rover positions together will transform the coordinates of all detail surveyed from the first point to the survey detail from the second point (*Figure 14-5, View B*).

Rural Area Boundary Surveys will require setting up the total station at a first point where one or more boundary markers can be seen. Fix the position with RTK and GPS. Orient to a second point, which is not yet fixed, and then measure angles and distances to the markers. Set up at the second point, fix the position, and orient to the first point. All previous measurements are transformed automatically via software of the total station. Survey the markers from the second point. Survey the boundary in this way using pairs or clusters of points (*Figure 14-5, View C*).

Utilities surveys involve the precise positioning of manholes, hydrants, distribution boxes, etc. for water, gas, and electricity installation. Buildings and trees along roads can prevent the use of optic and GPS rover equipment. To overcome these obstacles, set up the total station where GPS fixes are possible, such as at road intersections, open spaces, and even on the tops of buildings. Use pairs of fixed position setups as explained in the previous examples and then measure angles and distances to the objects that have to be surveyed (*Figure 14-5, View D*).

2.4.2 GPS Survey Planning Elements

Unlike conventional surveying technologies the Seabee's have generally not been using GPS systems long enough for the average EA surveyor to have amassed the "conventional wisdom" needed to reliably execute GPS surveys. One of the significant advantages of the GPS survey technique over conventional surveying techniques is that sites may be placed where they are required, irrespective of whether intervisibility between stations is preserved. Generally, the GPS stations would be "clustered" around the project focus, for example a road, power line corridor, etc. This is in contrast to traditional geodetic control, which was generally evenly spaced and the stations located in prominent locations such as at the tops of hills, to ensure that they were visible from afar. In addition, extra survey stations that "carry in" the control from the nearest geodetic control stations to the project area are not usually necessary for GPS work. Hence, even spacing of stations and selection of stations on the basis of terrain are no longer important considerations (*Figure 14-6*).

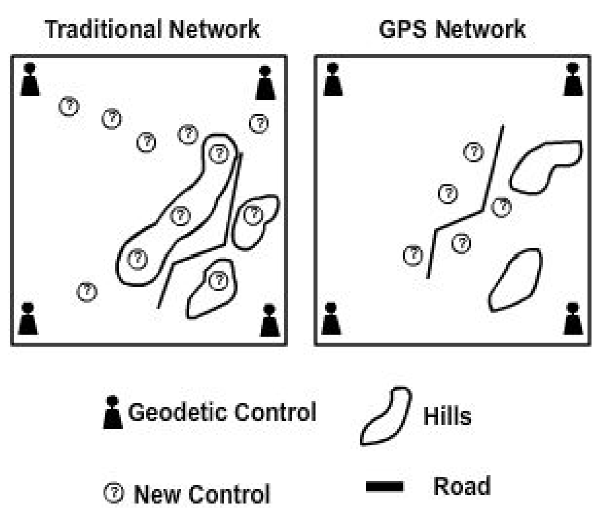


Figure 14-6 – Traditional vs. GPS control point plans

Once the number of GPS stations has been decided upon, and their approximate locations have been determined, other considerations may influence where additional stations are located, or where refinements to the network design could be made, such as the following:

- Some intervisible stations may need

to be included to define starting azimuths for subsequent conventional surveys. This may be best satisfied by providing some additional azimuth marks surveyed by GPS, rather than altering the original network design to incorporate station intervisibility. The reference marks may be set up a couple of hundred meters away (and, depending on the terrain, etc., perhaps up to a kilometer or more distant).

- There may be several existing stations in the area which could be included in the GPS survey. There are many reasons for occupying already established stations, from simple expediency, to necessary ties to previous work, to datum definition or the calibration or the calibration of GPS heights.
- There is generally no need to establish GPS stations to "connect" the main net to surrounding control unless distances are large. Although relative error is a function of interstation distance, it is not a linear relationship. For distances up to 20-30 km, an "ambiguity-fixed" solution is generally possible, and the accuracy of the position solution is of the order of 1 to 5 ppm. When, at longer distances, ambiguity resolution is not possible, the "ambiguity-free" solution is generally weaker by a factor of 2 to 3 compared to the "ambiguity-fixed" solution. It remains at this new (lower) relative accuracy for distances up to 100 km. The optimal solution for baselines longer than 100km (if using commercial GPS software) is one based on triple-differenced phase observables.
- Certain baselines may be designated as primary ones, for example, the baseline(s) defining the location of a project structure. These are critical to the final network and must be at both a higher accuracy and higher reliability. Hence they may need to be measured several times, or special instrumentation and procedures used. Network design may therefore not simply involve points, but baselines (or figures) as well.

2.4.3 GPS Network Shape

As in the case of conventional surveys, there is an impact arising from "structural" considerations, as some networks are superior to others with regard to "strength." Only independent baselines contribute to network strength, but GPS networks may have different shapes as in *Figure 14-7*, as well as different "strengths" arising from the number of independent baselines observed over a number of sessions.

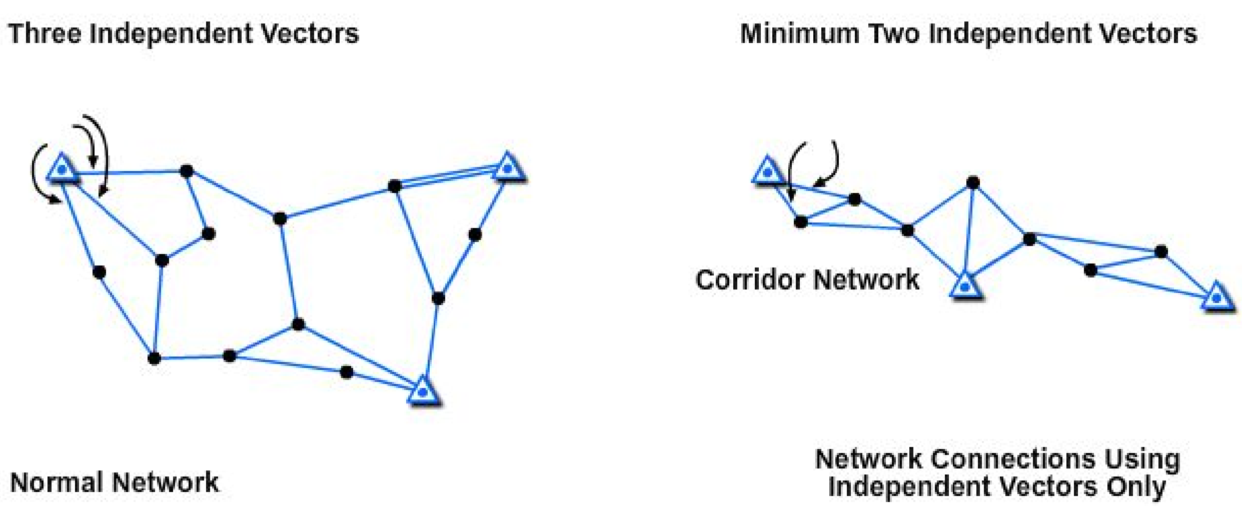


Figure 14-7 – GPS network shapes- "wide" and "narrow."

Test your Knowledge (Select the Correct Response)

4. **(True or False)** GPS satellites are in near-circular orbits around the globe at an altitude of approximately 12,400 miles above the earth.

   A. True
   B. False

5. Which, if any, of the following are two commonly referred to GPS transmit carrier frequencies?

   A. L1 and L2
   B. L3 and L4
   C. L6 and L7
   D. None of the above

6. What is the most important tool when using kinematic GPS surveying?

   A. Base station
   B. Total station
   C. Theodolite
   D. Rover

3.0.0 TRIANGULATION

In your previous studies and possibly on the job as part of a survey team, you learned that a principal method of locating points in horizontal control is traversing. As you know, traversing requires that distances and angles be measured at all stations. Another method, called ***triangulation,*** requires that distances be measured only at the beginning, at specified intervals, and at the end of the survey (*Figure 14-8*). Triangulation is also the most common type of geodetic survey. It differs from the plane survey in that more accurate instruments are used, instrumental errors are either removed, predetermined, or compensated for in the computations, and more rigorous procedures are employed to reduce observational errors. Another very important difference is that all of the positions established by triangulation are mathematically related to each other.

Both the triangulation method and the traverse method of control are based on the characteristics of the terrain and not on the degree of precision to be attained, that is, each system is equally precise under the conditions in which each is used. The examples of triangulation discussed in this chapter will be limited to triangles having sides less than 3,000 yards in length and to triangulation nets that do not extend more than 25,000 yards.

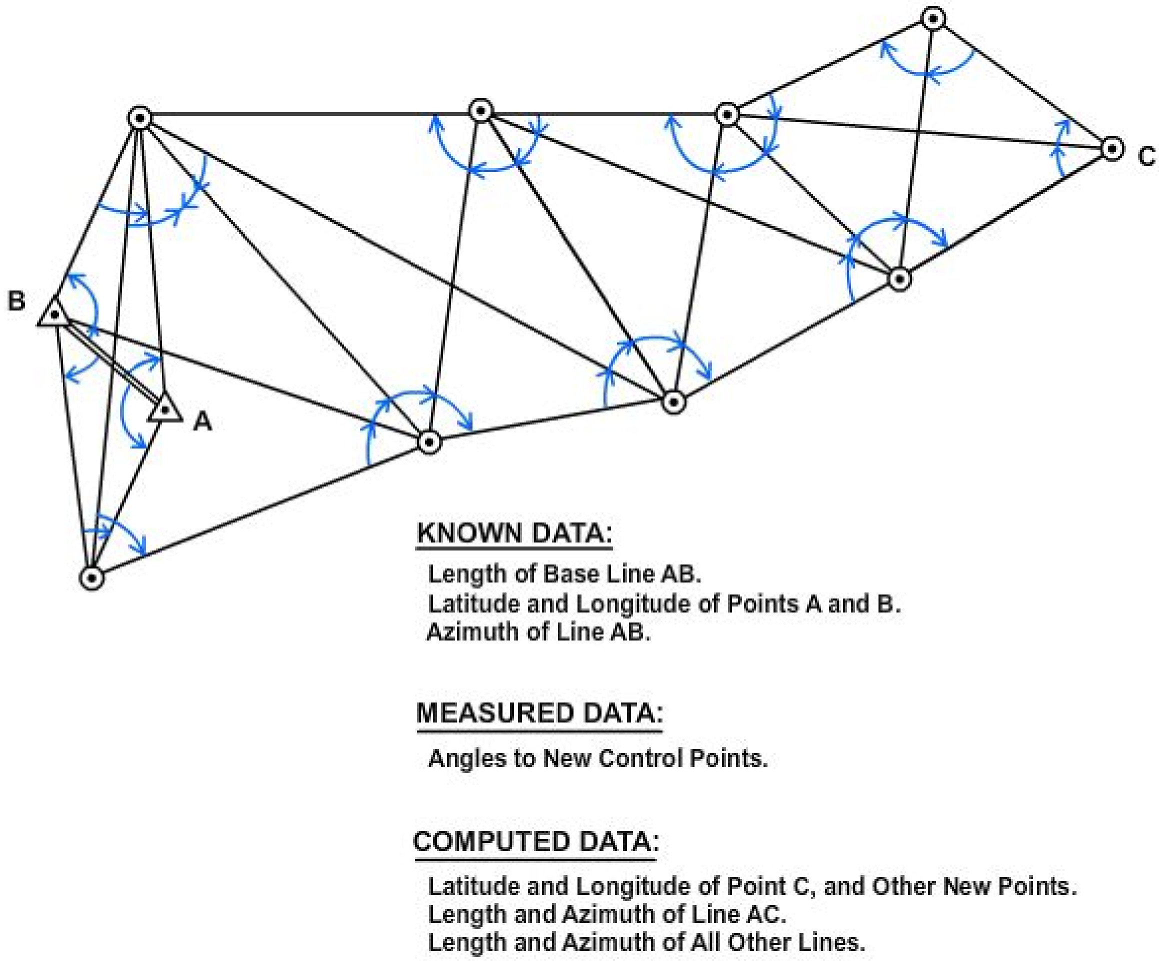


Figure 14-8 – Triangulation method.

The triangulation method is used principally in situations where the chaining of distances is impossible or infeasible except with the use of electronic measuring devices. Suppose as part of a survey you want to locate a point, which we will call point C, which is offshore; and the measured baseline, AB, is located on the shore. In this situation the triangulation method is used because the chaining of distances is impossible. The chaining of long distances, especially in rough country, also is not always possible; therefore, triangulation is used to establish horizontal control in large-area surveys.

In some large-area surveys conducted by triangulation, you must consider factors involving the curvature of the earth; hence, in such cases, ***geodetic*** triangulation is involved. Whether or not the curvature of the earth must be considered depends upon the area covered and the precision requirements of the survey.

The error resulting in horizontal measurements when you ignore the curvature of the earth amounts to about 1 foot in 34 1/2 miles. This means that in most ordinary surveying, an area of 100 square miles may be plane-triangulated without significant error. With respect to surveying performed in support of Seabee construction projects, we are concerned with plane triangulation only. For information concerning geodetic triangulation, you should refer to commercial publications.

There are three types of triangulation networks that can be used when conducting a triangulation survey. From the primary and secondary triangulation stations, though the types of signals used in marking triangulation stations, to the checking for precision and locations of points within triangulation networks. When performed correctly, they are an efficient means of accurately plotting a construction site.

3.1.0 Supervision and Triangulation Surveys

In triangulation surveys, the duties of the EA are those of party chief, that is, directing the triangulation survey. By keeping the triangulation notes and being at the spot where any important measurement is made, you can verify the readings personally. The party chief is responsible for selecting triangulation stations and erecting triangulation signals and towers as needed. You will also determine the degree of precision to be attained. The party chief is also responsible for performing the computations necessary to determine the horizontal locations of points in the triangulation system by bearing and distance.

Triangulation is used extensively as a means of control for topographic and similar surveys. A triangulation system consists of a series of triangles. At least one side of each triangle is also a side of an adjacent triangle; two sides of a triangle may form sides of adjacent triangles. By using the triangulation method of control, you will not need to measure the length of every line. However, two lines are measured in each system, one line at the beginning and one at the closing of the triangulation system. These lines are called base lines and are used as a check against the computed lengths of the other lines in the system. The recommended length of a base line is usually 1/6 to 1/4 of that of the sides of the principal triangles. The transcontinental system established by the National Geodetic Survey is an example of an extensive high-order triangulation network to establish control across the United States.

3.2.0 Types of Triangulation Networks

In triangulation there are three types of triangulation networks (or nets): the chain of single triangles, the chain of polygons, and the chain of quadrilaterals.

3.2.1 Chain of Single Triangles

The simplest triangulation system is the chain of single triangles *Figure 14-9*. Suppose *AB* is the base line and measures 780.00 feet in length. Suppose, also, that angle *A* (that is, the observed angle *BAC)* measures 98°54´ and that angle *ABC* measures 32°42´. (In actual practice you will use more precise values than these; we are using rough values to simplify the explanation.) Subtracting the sum of these two angles from 180°, we get 48°24´ for angle *ACB.*

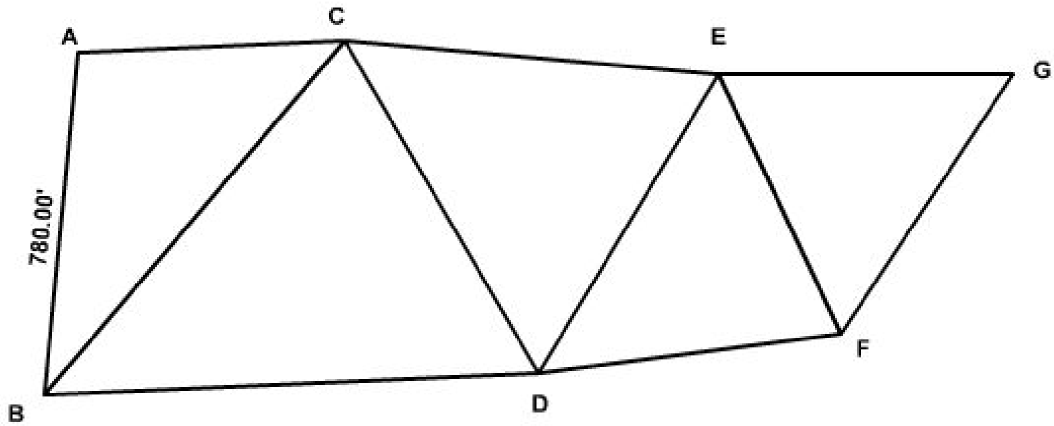


Figure 14-9 – Chain of single triangles.

Next, solve for sides *BC* and *AC* by using the law of sines as follows:

$$BC = AB\frac{\sin A}{\sin C} \qquad AC = AB\frac{\sin B}{\sin C}$$

$$= AB\frac{\sin 98^{\circ}54'}{\sin 48^{\circ}25'} \qquad = AB\frac{\sin 32^{\circ}42'}{\sin 48^{\circ}24'}$$

$$= 1030.50\,ft \qquad = 563.50\ ft$$

Now that you know how to find the length of *BC,* you can proceed in the same manner to determine the lengths of *BD* and *CD.* Knowing the length of *CD,* you can proceed in the same manner to determine the lengths of *CE* and *DE,* knowing the length of *DE,* you can determine the lengths of *DF* and *EF,* and so on. You should use this method only when locating inaccessible points, not when a side of the triangle is to be used to extend control.

In comparison with the other triangulation systems, the chain of single triangles has two disadvantages. First, it can be used to cover only a relatively narrow area. Second, it provides no means for cross-checking computed distances using computations made by a different route.

In *Figure 14-9*, for example, the only way to compute the length of *BC* is by solving the triangle *ABC,* the only way to compute the length of *CD* is by solving the triangle *BCD* (using the length of *BC* previously computed), and so on. In the systems about to be described, a distance maybe computed by solving more than one series of triangles.

3.2.2 Chain of Polygons

Technically speaking, of course, a triangle is a polygon, and therefore a chain of single triangles could be called a chain of polygons. However, in reference to triangulation figures, the term ***chain of polygons*** refers to a system in which a number of adjacent triangles are combined to form a polygon (*Figure 14-10*). Within each polygon the common vertex of the triangles that compose it is an observed ***triangulation station*** (which is not the case in the chain of quadrilaterals described later).

You can see how the length of any line shown can be computed by two different routes. Assume that *AB* is the base line, and you wish to determine the length of line *EF.* You can compute this length by solving triangles *ADB, ADC, CDE,* and *EDF,* in that order, or by solving triangles *ADB, BDF,* and *FDE,* in that order. You can also see that this system can be used to cover a wide territory. It can cover an area extending up to approximately 25,000 yards in length or breadth.

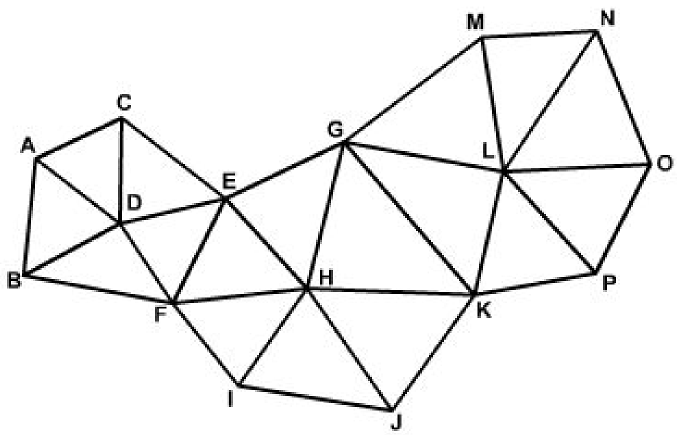
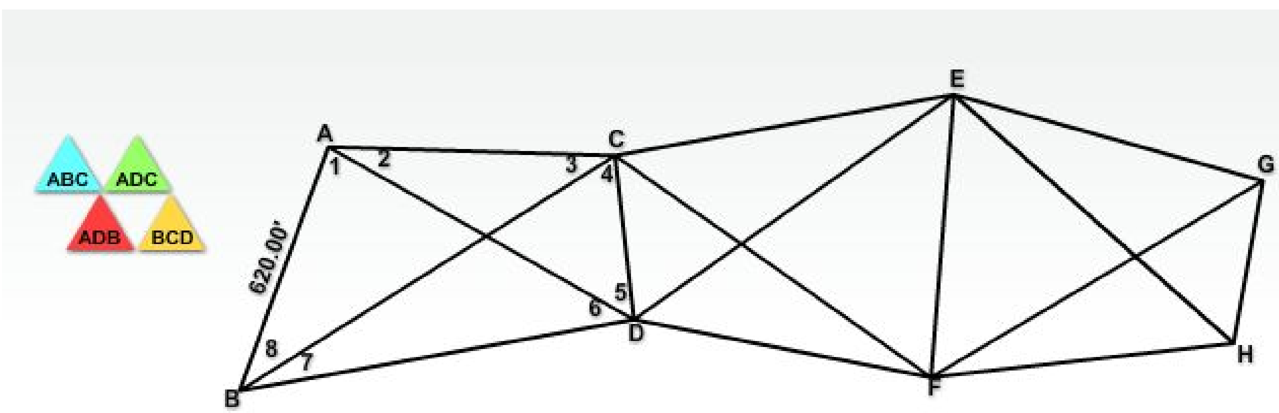


Figure 14-10 — Chain of polygons.

3.2.3 Chain of Quadrilaterals

A quadrilateral, too, is technically a polygon, and a chain of quadrilaterals would be technically a chain of polygons. However, with reference to triangulation figures, the term ***chain of quadrilaterals*** refers to a figure arrangement like that shown in *Figure 14-11*.



In quadrilateral ACDB, there are four overlapping triangles as follows: *ADC, ADB, ABC*, and *BCD.*
Solving these four triangles will give you two computations for the length of each unknown side of the quadilateral.

Figure 14-11 – Chain of quadrilaterals.

Within each of the quadrilaterals shown, the triangles on which computations are based are not the four adjacent triangles visible to the eye, but four overlapping triangles—each of which has sides that form two sides of the quadrilateral and a cross cutting diagonal of the quadrilateral. For example, in quadrilateral *ACDB* there are four overlapping triangles as follows: *ADC, ADB, ABC,* and *BCD.* You can see that solving these four triangles will give you two computations for the length of each unknown side of the quadrilateral.

Consider, for example, the quadrilateral *ACDB.* Look at angle *BAC.* We will call the whole angle at a comer by the letter (as, angle A) and a less-than-whole angle at a corner by the number shown (as, angle 1). The angles at each station on the quadrilateral, as measured with a protractor to the nearest 0.5 degree and estimated to the nearest 0.1 degree, are sized as follows in *Table 14-2*:

Table 14-2 – Angles at Each Station on the Quadrilateral.

Angle	Size	Angle	Size
1	79° 06′	5	53° 30′
2	29° 00′	6	40° 24′
3	34° 06′	7	22° 42′
4	63° 06′	8	37° 48′

The angles that make up each of the four overlapping triangles, together with their natural sines, are as follows in *Table 14-3*:

Table 14-3 – Four Overlapping Triangles with Their Natural Sines

Triangle	Angle	Size	Sine
ABC	A	108° 06′	0.950516
	3	34° 06′	0.560639
	8	37° 48′	0.612907
ADB	B	60° 30′	0.870356
	1	79° 06′	0.981959
	6	40° 24′	0.648120
ADC	C	97° 30′	0.991445
	2	29° 00′	0.484810
	5	53° 30′	0.803857
BCD	D	93° 54′	0.997684
	4	63° 24′	0.894154
	7	22° 42′	0.385906

Note that the total sum of the angles is 360°, which it should be for a quadrilateral, and that the sum of the angles in each triangle is 180°, which is also geometrically correct.

To solve the quadrilateral, you solve each of the overlapping triangles. First, you solve triangle *ABC* for sides *AC* and *BC,* using the law of sines as follows:

$$AC = AB\frac{\sin 8}{\sin 3} = AB\frac{\sin 37°48'}{\sin 34°06'} = 677.80\ ft$$

$$BC = AB\frac{\sin A}{\sin 3} = AB\frac{\sin 108°06'}{\sin 34°06'} = 1{,}051.16\ ft$$

Then, using similar computation procedures, you then solve triangle *ABD* for sides *BD* and *AD,* triangle *ADC* for *AC* and *CD,* and triangle *BCD* for *BD* and *CD.* The solutions for each of the overlapping triangles are summarized as follows:

Table 14-4 – Solutions of the Overlapping Triangles.

Triangle	Side	Length
ABC	AC	677.80
	BC	1,051.16
ADB	BD	939.35
	AD	832.59
ADC	AC	675.06
	CD	07.13
BCD	BD	942.08
	CD	406.59

As you can see, for each of the unknown sides of the quadrilateral *(AC, CD,* and *BD),* values have been obtained by two different routes. You can also see that there are discrepancies in the values, almost the same for *AC* and *BD* and smaller for *CD.* All the discrepancies shown are much larger than would be tolerable in actual practice; they reflect the high imprecision of the original protractor measurement of the angles. The example has been given here only to illustrate the basic principles and procedures of chain-of-quadrilateral triangulation.

Later in this chapter you will see how observed angles (measured in the field with the required precision) are adjusted to ensure that values computed by different routes will be practically close enough to each other to satisfy precision requirements.

3.3.0 Triangulation Stations, Signals, and Instrument Supports

All triangulation stations of third order or higher must be identified on the ground with a station marker, at least two reference markers, and if necessary an azimuth marker. These markers are usually embedded in or etched on a standard station monument. Station markers, monuments, and station referencing are discussed in EA Basic, Chapter 12. For low-order surveys, unless otherwise required, the stations may be marked with 2-inch by 2-inch wooden hubs.

A primary triangulation station is both a sighted station and an instrument station, that is, it is a point sighted from other stations and also a point where an instrument is set up for sighting other stations. A secondary triangulation station is one that is sighted from primary stations but is not itself used as an instrument station. Only the primary stations are used to extend the system of figures.

Each triangulation station must be marked in a way that will make it visible from other stations from which it is sighted. A mark of this kind is called a triangulation signal**.** For a secondary station, the signal may be relatively simple, such as a pole set in the ground or in a pile of rocks, or a pole set on the ground and held erect by guys. An object already in place, such as a flag pole, a church spire, or a telegraph pole, will serve the purpose. When the instrument itself must be elevated for visibility, a tower is used.

3.3.1 Targets

A target is generally considered to be a non-illuminating signal. Target requirements can be met by three general types—tripods, bipeds, and poles—all of which may incorporate variations. The targets are constructed of wood or metal framework with cloth covers.

For a target to be easily visible against both light and dark backgrounds, it should be constructed in alternating belts of red and white or red and yellow. For ready bisection, it should be as narrow as possible without sacrificing distinctness. A target that subtends an angle of 4 to 6 seconds of arc will fulfill this purpose. Since 1 second of arc equals 0.5 centimeters at a 1-kilometer distance, an angle of 6 seconds requires a target 3 centimeters wide at 1 kilometer or 30 centimeters at 10 kilometers. Under adverse lighting conditions, the target width will have to be increased. Flags of an appropriate size may be added to aid in finding the target. All cloth used on targets should be slashed after construction to minimize wind resistance.

3.3.1.1 Types of Targets

The tripod target is the most satisfactory from the standpoint of stability, simplicity of construction, durability, and accuracy. It ranges from a simple hood of cloth, cut and sewn into a pyramid shape and slipped over the instrument tripod, to the permanent tripod with the legs embedded in concrete, sides braced, a vertical pole emplaced, and the upper part boarded up and painted. Temporary tripod targets may be constructed of 2-inch by 2-inch lumber, pipes, poles, or bamboo joined at one end by wire or bolts threaded through drilled holes. The tripod must be well guyed and plumbed as depicted in *Figure 14-12,* and the legs should be set in depressions to prevent lateral movement. On uneven ground one leg may have to be shortened or dug in to maintain a symmetrical appearance from all directions. Signal cloth wrapped around the tripod should be used only on low-order (fourth-order) work as it is almost impossible to make it symmetrical around the station.

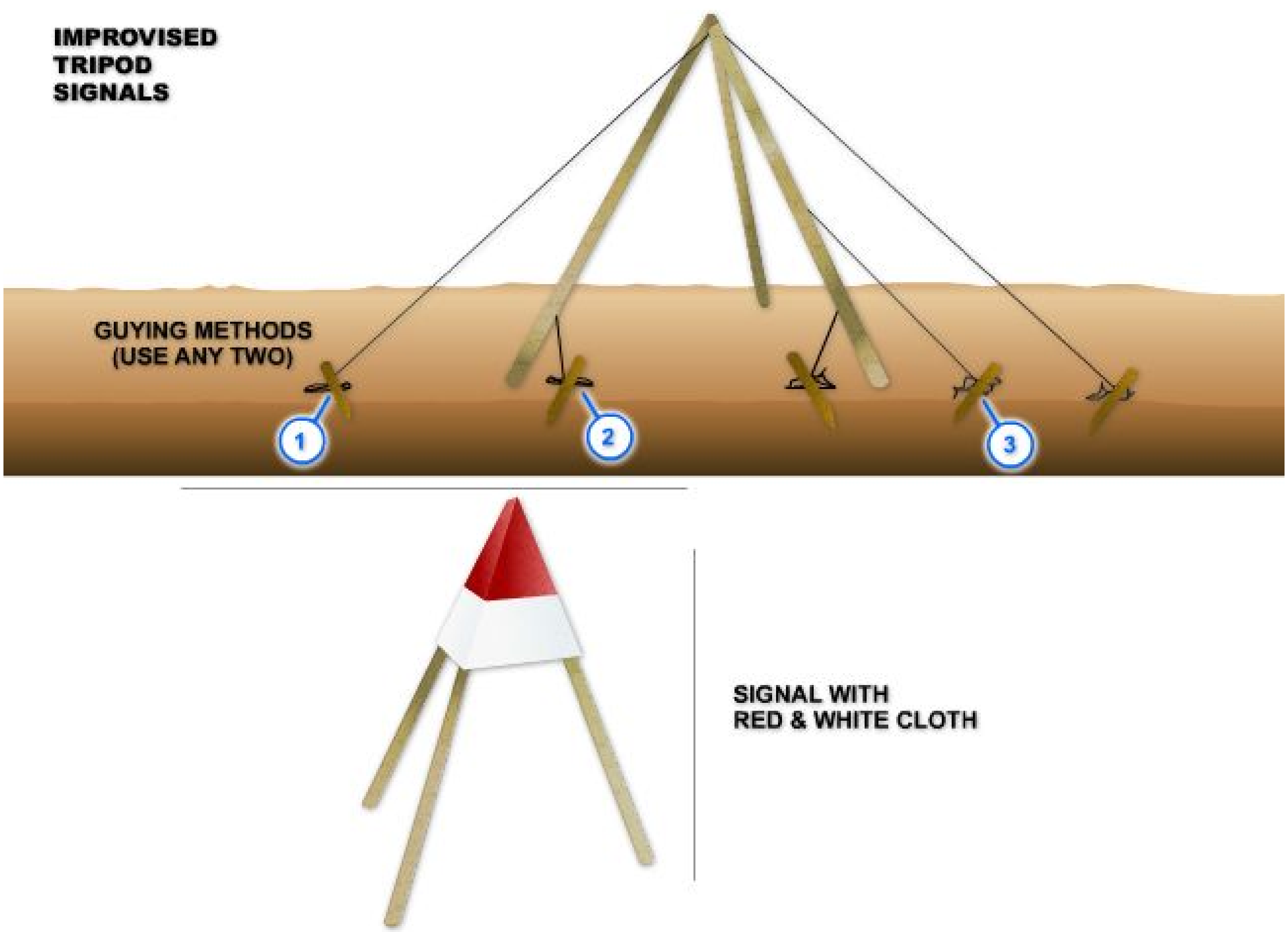


Figure 14-12 – Tripod targets.

Bipod targets are more simply constructed than tripods but are less stable and must be strongly guyed. *Figure 14-13* shows a standard surveying bipod target. It is carried disassembled in a canvas case about 53 inches long. It can be assembled, erected, and plumbed by two personnel in 15 minutes. If this target must be left standing in the weather for any extended period, the rope guys should be replaced with wire and two more wire guys added to each end of the crossbar. In soft ground the pointed legs will sink unevenly because of wind action and rain; therefore, they should be set in holes bored in the end of wooden stakes driven flush or in a short piece of 2-inch by 4-inch lumber laid flat in a shallow hole.

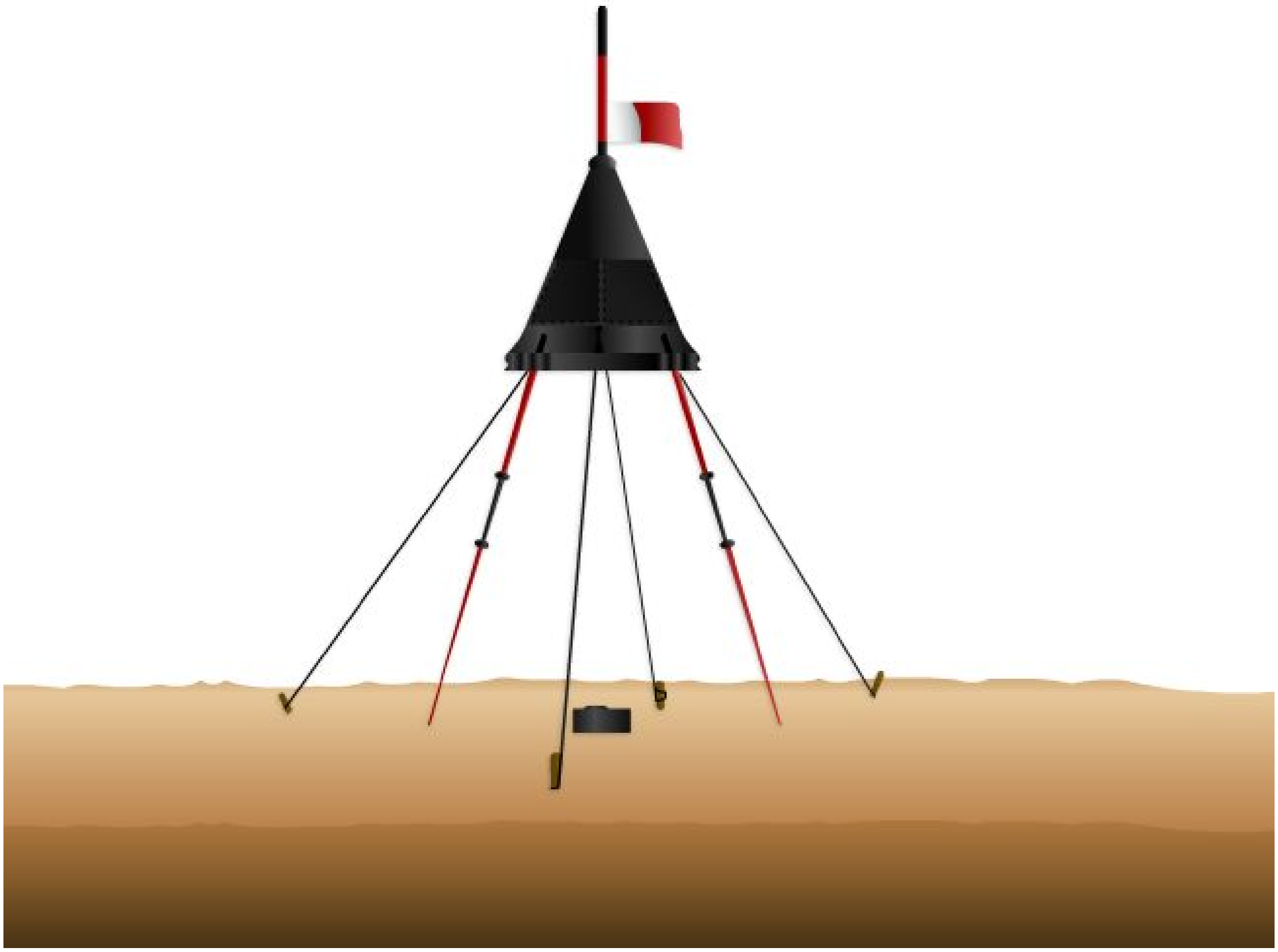


Figure 14-13 – Bipod targets.

Pole targets such as in *Figure 14-14* are seldom used because the station cannot be occupied while the target is in place. In certain cases, as when an unoccupied station must be sighted and cutting of lines of sight is difficult or impossible, a pole target that can be seen above the trees maybe erected. The staff may be constructed of 2-inch by 2-inch lumber or cut poles, varying from about 2 inches to 6 inches in diameter. The method of joining sections of 2-inch by 2-inch lumber and the construction of a panel target are shown in *Figure 14-14*. The targets must be plumbed by manipulation of the guy wires. Special care must be taken when warped or crooked boards are used to construct pole targets, and they must be checked for eccentricity.

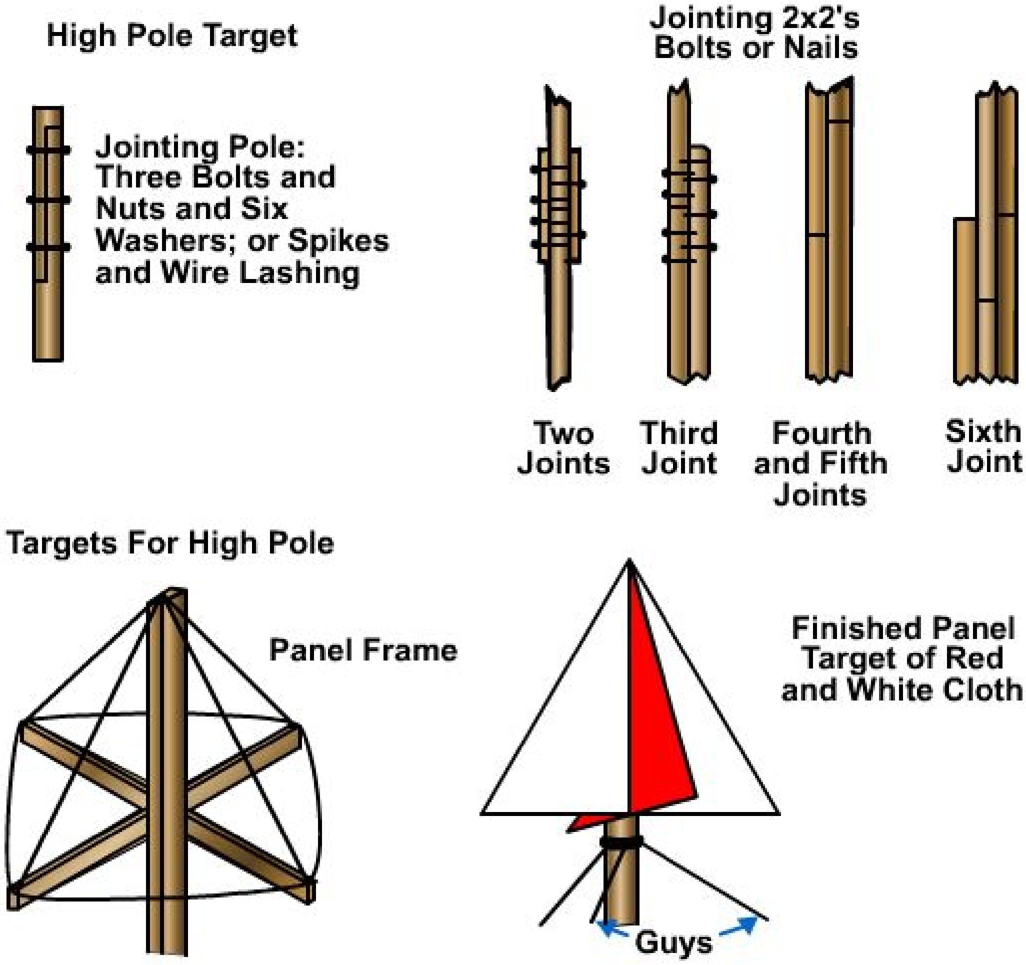


Figure 14-14 – Pole targets.

3.3.2 Signals

Signals are those survey targets that either are illuminated by natural sunlight or are electrically lighted by use of wet or dry cell batteries. The observations for all first- and second-order triangulation and first-order traverse are usually done at night using signal lights, because of more stable atmospheric conditions, which allow for better pointings. Observations may be made during daylight hours using lights, but for high-accuracy surveys, this is done only under extreme conditions.

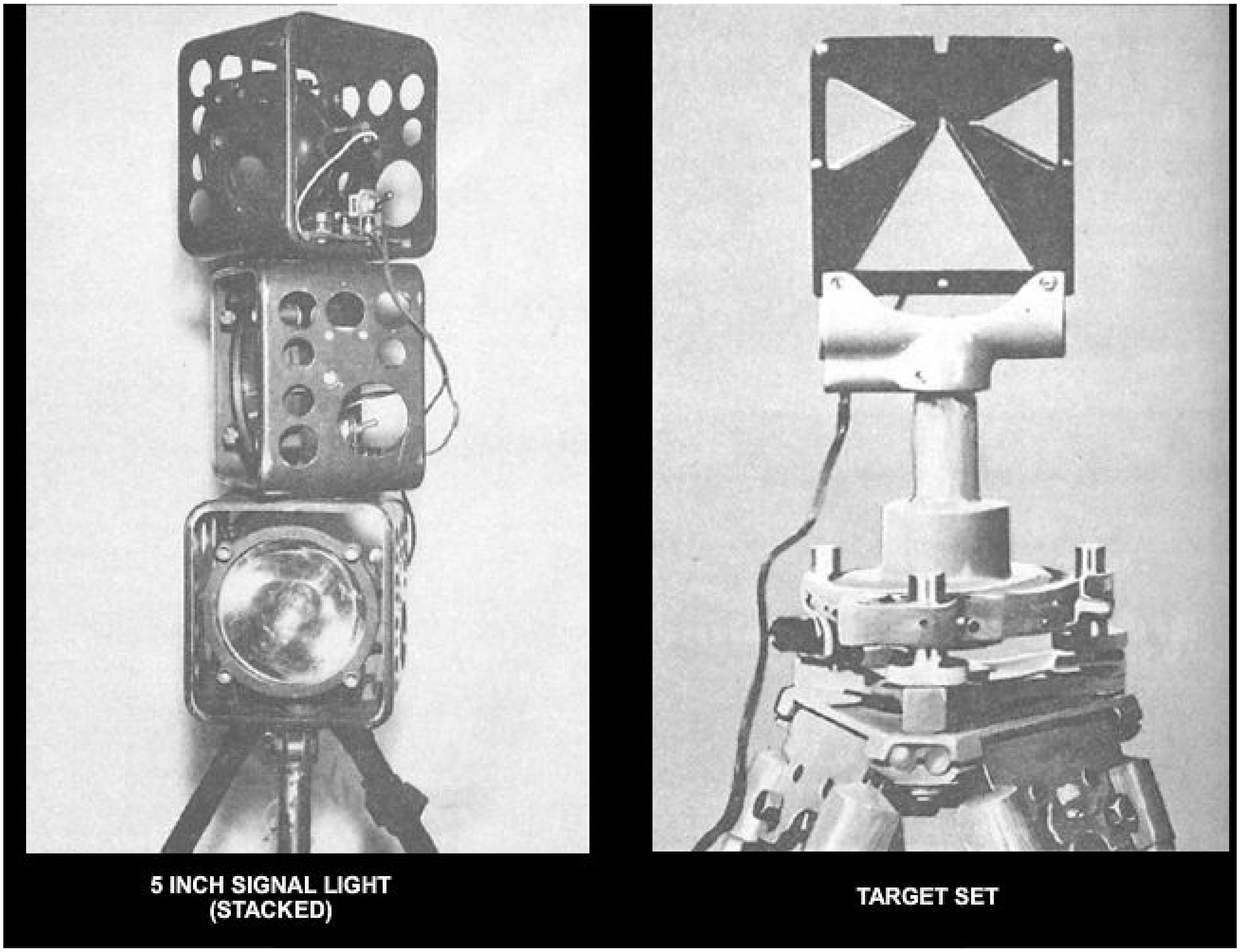


Figure 14-15 – Signaling devices.

Some examples of signals are signal lights and a target set (*Figure 14-15)* and a heliotrope (*Figure 14-16*). The target set is a precise lighting device that is generally used for short traverse lines. The heliotrope is a device that reflects the rays of the sun through a pair of mirrors set over a point and toward an observer on another station. When standard signals are not available, expedient lights can be used. Examples of expedient lights are the headlights of a vehicle, a masked lantern, or a boxed light bulb.

Figure 14-16 – Heliotrope and case.

3.3.3 Supports (Towers)

Towers must be built on some stations to raise the lines of sight to clear obstructions or to lengthen the lines of sight to increase distances between stations of area surveys. A tower consists of an instrument stand (inner structure) and a platform to support the observer (outer structure). Towers fall roughly into three classes: prefabricated aluminum or steel, wooden, and expedient towers. The towers are usually constructed by a separate crew, whose size depends upon the type of tower being built. The expedient tower is usually a tower or high structure that is already in the area. Two examples of towers are shown in *Figures 14-17* and *14-18*.

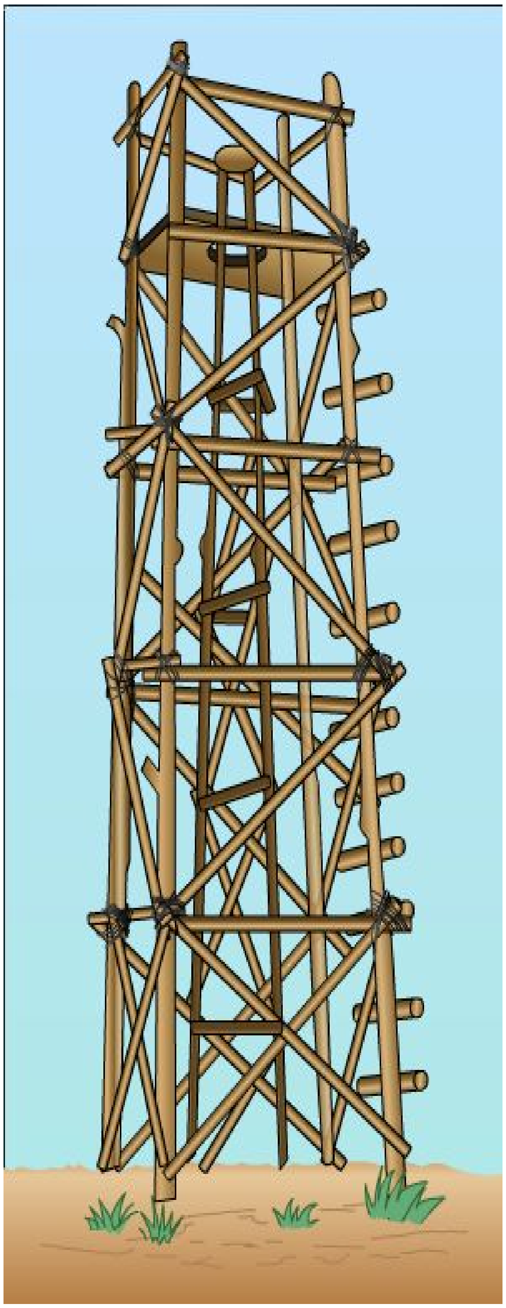


Figure 14-17 — Pole tower.

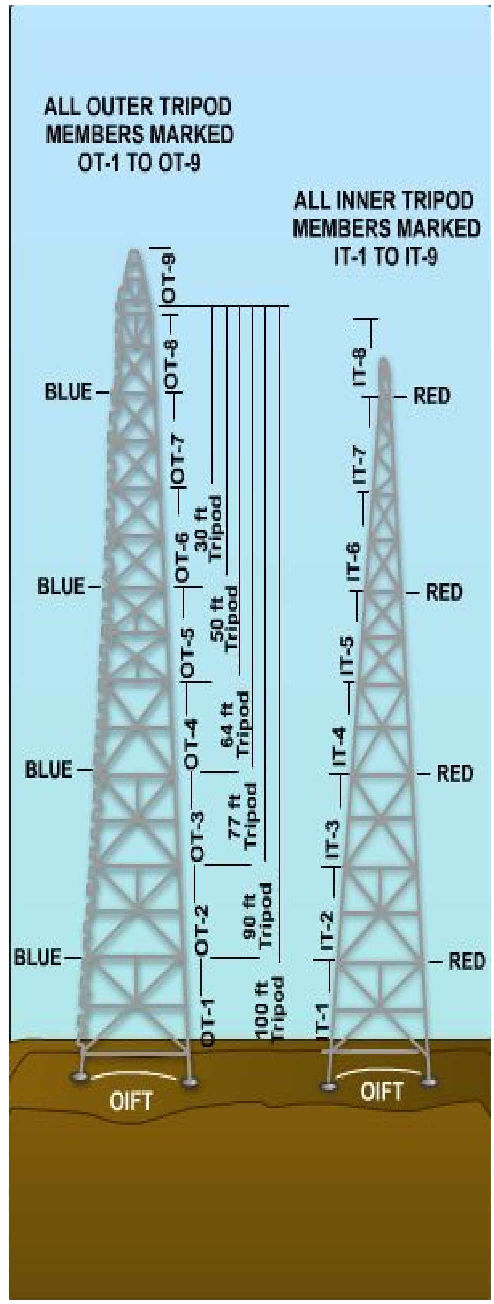


Figure 14-18 — Expedient tower.

3.4.0 Triangulation Procedures

First, a reconnaissance is made to determine the best locations for the stations. Next, the necessary tower, stands, or masts are erected to make the stations intervisible, and the stations are marked. Then, angles at specified stations are carefully measured with a theodolite. Any distances needed for the control of the triangulation, or any lengths required are measured next.

3.4.1 Reconnaissance

The first consideration with regard to the selection of stations is, of course, ***intervisibility*****.** An observation between two stations that are not intervisible is impossible. Next is accessibility**.** Obviously, a station that is inaccessible cannot be occupied, and deciding between two stations which are otherwise equally feasible, you should chose the one that provides the easier access.

The next consideration involves strength of figure**.** In triangulation, the distances computed (that is, the lengths of triangle sides) are computed by way of the law of sines. The more nearly equal the angles of a triangle are, the smaller the ratio of error in the sine computations. The ideal triangle, then, would be one in which each of the three angles measured 60°; this triangle would, of course, be both equiangular and equilateral for a total of combined angles of 180°.

Values computed from the sines of angles near 0° or 180° are subject to large ratios of error. As a general rule, you should select stations that will provide triangles in which no angle is smaller than 30° or larger than 150°.

3.4.2 Signal Erection

After the stations have been selected, the triangulation signals or triangulation towers should be erected. When you erect triangulation towers or signals, remember that it is imperative for these stations to be intervisible. It is also important that the target be large enough to be seen at a distance, that is, the color of the target must be selected for good visibility against the background where it will be viewed. When observations are made during daylight hours with the sun shining, a heliotrope is a very effective target. When triangulation surveys are made at night, lights must be used for targets. Therefore, target sets with built-in illuminations are very effective.

3.4.3 Measurement of Angles

The precision with which angles in the system are measured will depend on the ***order of precision*** prescribed for the survey. The precision of a triangulation system may be classified according to (1) the average error of closure of the triangles in the system and (2) the ratio of error between the measured length of a base line and its length as computed through the system from an adjacent base line. Large government triangulation surveys are classified in precision categories as follows:

Table 14-5 – Precision Categories.

Order of Precision	Triangle Avg. Closure (in Seconds)	Base Line Ratio
First	1	1: 25,000
Second	3	1: 10,000
Third	5	1: 5,000

For third-order precision, angles measured with a 1-minute transit will be measured with sufficient precision if they are repeated six times. As explained in EA Basic, Chapter 19, six repetitions with a 1-minute transit measures angles to the nearest 5 seconds. To ensure elimination of certain possible instrumental errors, you should make half of the repetitions with the telescope erect and half with the telescope reversed. In each case, the horizon should be closed around the station.

3.4.4 Determination of Direction

As you learned earlier in this chapter, most astronomical observations are made to determine the true meridian from which all azimuths are referred. In first-order triangulation systems, these observations are used to determine latitude and longitude. Once the true meridian is established, the azimuths of all other sides are computed from the true meridian.

To compute the coordinates of triangulation stations, you must determine the latitudes and departures of the lines between stations; to do this, you must determine the directions of these lines. The latitude of a traverse line means the length of the line as projected on the north-south meridian running through the point of origin. The departure of a traverse line means the length of the line as projected on the east-west parallel running through the point of origin.

3.4.5 Base Line Measurement

The accuracy of all directions and distances in a system depends directly upon the accuracy with which the length of the base line is measured. Therefore, base line measurement is vitally important. You must use a transit to give precise alignment while measuring a base line. For third-order triangulation measurement with a steel tape, you are required to incorporate all the tape corrections described in the EA Basic, Chapter 13. For measurement over rough terrain, end supports for the tape must be provided by posts driven in the ground or by portable tripods. These supports are usually called chaining bucks. The slope between bucks is determined by measuring the difference in elevation between the tops of the bucks with a level and rod.

On the top of each buck, a sheet of copper or zinc is tacked down, which provides a surface on which tape lengths can be marked. Bucks are set up along the base line at intervals of one-tape length. The tape, with thermometers fastened at each end, is stretched between the supports and brought to standard tension by a tensiometer (spring balance). When the proper tension is indicated, the position of the forward end is marked on the metal strip with a Marking Awl or some other needle-pointed marker. At the same time, the thermometer readings are taken.

If stakes driven at tape-length intervals are used as tape supporters, the end of the tape may begin to lie slightly off the metal marking strip on the buck after a few tape intervals have been laid off. To take care of this situation, the head chairman carries a finely divided (to 0.001 ft) pocket scale. With this scale he or she measures the distance that the tape must be set back or set forward to bring the end again on the marking strip. The set back or set forward is entered in the field notes and deducted from or added to the tape length for that particular interval.

Figure 14-19 shows field notes for a base line measurement. In this case the tape was supported on stakes driven at full-tape, 100-foot intervals. With the exception of the interval between stakes 5 and 6, the horizontal distance between each adjacent pair of stakes amounts to the standard tape length (with the tape supported at both ends, and with standard tension applied), as corrected for temperature and for slope. For the interval between stakes 5 and 6 (where there is, as you can see, a forward set), the horizontal distance amounts to the standard tape length plus 0.104 foot, as corrected for temperature and for slope. The length of the base line will, of course, amount to the sum of the horizontal distances.

MEASUREMENT OF EAST BASE LINE

From Stake NO.	To Stake NO.	Thermometers #1 Deg. F	Thermometers #2 Deg. F	Set Fwd Ft	Set Back Ft
1	2	73.0	72.5		
2	3	73.0	73.0		
3	4	73.5	73.0		
4	5	72.5	72.5		
5	6	72.5	73.0	0.104	
6	7	73.0	73.5		
7	8	73.5	73.5		
8	9	73.5	73.0		

3 June 1994	Smith, J. EN Recorder
Cloudy,Warm	Jones, R., EA. S Marker
Measuring Forward	Brown, B., CN, H. Chr
100-Ft Tape #2	Adams, B., CN, R. Chr
Tensionmeter #4	
Tape Supported at 0 & 100 Ft	
Thermometers #11 & #2	

Figure 14-19 – Field notes for a base line measurement.

Note that in this case the line is being measured forward. After the forward measurement, the line is again measured in the backward direction. If the backward measurement varies slightly from the forward measurement, the average is taken as the length of the base line. A large discrepancy would, of course, indicate a mistake in one measurement or the other.

Rather than using chaining operations to perform base line measurements, an electronic distance meter (EDM) can be used. The use of EDM equipment greatly simplifies the measurement of base lines in triangulation.

3.4.6 Computations

In triangulation of ordinary precision or higher, the observed angles are adjusted before the lengths of the triangle sides are computed. The most rigorous and accurate of adjustment methods is the ***least squares method***, which involves the computation of the most probable values of the adjusted quantities. In many advanced surveying textbooks, the least squares method is preferred; however, calculation of the probable values of the unknowns involves a level of mathematics (calculus) that is beyond that required of the Engineering Aid. Therefore, in this text we will discuss more elementary adjustment procedures that while less accurate than the method of least squares, yield satisfactory results.

There are two steps in angle adjustment, called station adjustment and figure adjustment. Station adjustment applies the fact that the sum of the angles around a point is 360°. Figure adjustment applies the fact that the sum of the interior angles of a polygon is $(n - 2) \times 180°$, with n representing the number of sides of the polygon.

3.4.7 Station Adjustment

Station adjustment applies the fact that the sum of the angles around a point is 360°. Figure adjustment applies the fact that the sum of the interior angles of a polygon is $(n - 2) \times 180°$, with n representing the number of sides of the polygon.

3.4.7.1 Adjusting a Chain of Triangles

In station adjustment you compute the sum of the measured angles around each station, determine the extent to which it differs from 360°, and distribute this difference over the angles around the station according to the number of angles.

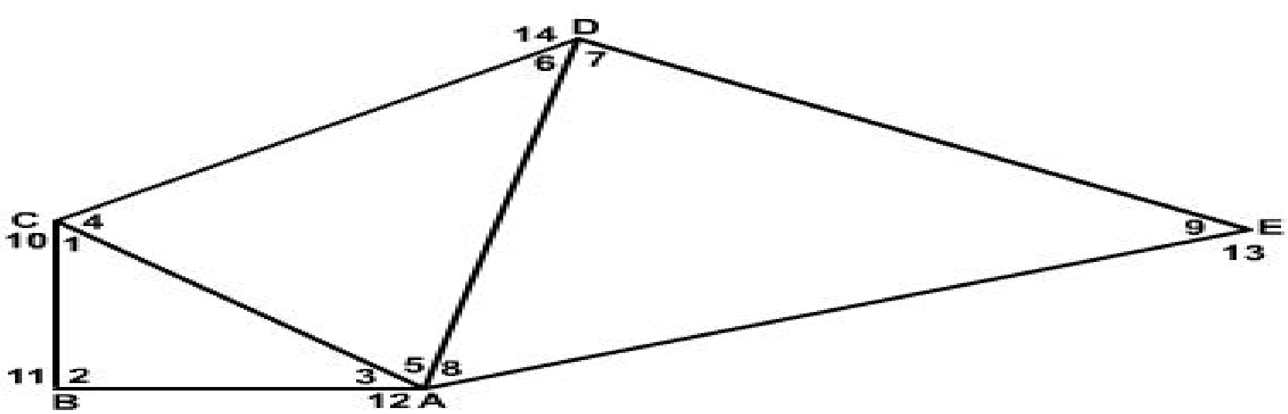


Figure 14-20 — Chain of triangles.

Table 14-6 – Station Adjustment for Chain of Triangles.

Station	Angle	Observed Value (6 Repetitions)			Value Adjusted for Station		
A	3	41°	02′	02″	41°	01′	56″
	5	61°	10′	41″	61°	10′	35″
	8	56°	08′	48″	56°	08′	42″
	12	201°	38′	54″	201°	38′	47″
	Sum	360°	00′	25″	360°	00′	00″
B	2	92°	47′	30″	92°	47′	34″
	11	267°	12′	21″	267°	12′	26″
	Sum	359°	59′	51″	360°	00′	00″
C	1	46°	10′	12″	46°	10′	10″
	4	75°	31′	02″	75°	31′	00″
	10	238°	18′	52″	238°	18′	50″
	Sum	360°	00′	06″	360°	00′	00″
D	6	43°	18′	19″	43°	18′	20″
	7	74°	43′	03″	74°	43′	05″
	14	241°	58′	33″	241°	58′	35″
	Sum	359°	59′	55″	360°	00′	00″
E	9	49°	07′	58″	49°	07′	58″
	13	310°	52′	01″	310°	52′	02″
	Sum	359°	59′	59″	360°	00′	00″

Figure 14-20 shows a chain of triangles. Station adjustment for this chain of triangles is given in *Table 14-6*. As you can see, at station *A* the sum of the observed interior angles 3, 5, and 8 plus the observed exterior closing angle 12 comes to 360°00´25´´. This differs from 360° by 25 seconds. The number of angles around the station is four; therefore, the correction for each angle is one fourth of 25, or 6 seconds, with 1 second left over. The sum of the observed angles is in excess of 360°; therefore, 6 seconds was subtracted from the observed value of each interior angle and 7 seconds from the observed value of the exterior angle. The angles around the other stations were similarly adjusted, as shown.

3.4.8 Figure Adjustment

The next step is the figure adjustment for each of the triangles in the chain. For a triangle, the sum of the interior angles is 180°. The figure adjustment for each of the three triangles illustrated in *Figure 14-20* is shown in *Table 14-7*.

Table 14-7 – Figure Adjustment for Chain of Triangles.

Triangle	Angle	Value after Station Adjustment			Value after Figure Adjustment		
ABC	1	46°	10′	10″	46°	10′	16″
	2	92°	47′	34″	92°	47′	41″
	3	41°	01′	56″	41°	02′	03″
	Sum	179°	59′	40″	180°	00′	00″
ACD	4	75°	31′	00″	75°	31′	02″
	5	61°	10′	35″	61°	10′	37″
	6	43°	18′	20″	43°	18′	21″
	Sum	179°	59′	55″	180°	00′	00″
ADE	7	74°	43′	05″	74°	43′	10″
	8	56°	08′	42″	56°	08′	47″
	9	49°	07′	58″	49°	08′	03″
	Sum	179°	59′	45″	180°	00′	00″

As you can see, the sum of the three adjusted observed interior angles in triangle *ABC* (angles 1, 2, and 3) comes to 179°59′40″. This is 20 seconds less than 180°, or 20/3, or 6 seconds for each angle, with 2 seconds left over. Therefore, 6 seconds was added to station adjusted value of angle 1, and 7 seconds each was added to the measured values of angles 2 and 3. The angles in the other two triangles were similarly adjusted.

3.4.9 Adjusting a Chain of Quadrilaterals

The station adjustment for a chain of quadrilaterals is the same as that for a chain of triangles. The next step is a figure adjustment like that for a chain of triangles, except that the sum of the interior angles of a quadrilateral is (4 - 2) 180°, or 360°.

Next for a quadrilateral, comes another figure adjustment based on the four overlapping triangles within the quadrilateral. To understand this figure adjustment, study the quadrilateral shown in *Figure 14-21*. The diagonals in this quadrilateral intersect to form vertically opposite angles 9-10 and 11-12. From you knowledge of geometry, you know that when two straight lines intersect the vertically opposite angles thus formed are equal. From the fact that the sum of the angles in any triangle is 180°, it follows that for any pair of vertically opposite angles in *Figure 14-21*, the sums of the other two angles in each of the corresponding triangles must be equal.

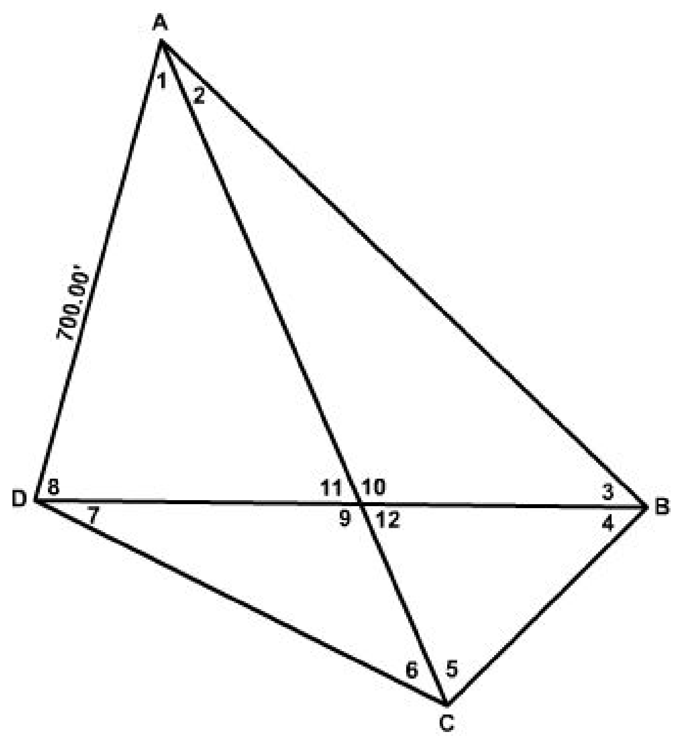


Figure 14-21 ─ Quadrilateral.

For example: In *Figure 14-21*, angles 11 and 12 are equal vertically opposite angles. Angle 11 lies in a triangle in which the other two angles are angles 1 and 8; angle 12 lies in a triangle in which the other two angles are angles 4 and 5. It follows, then, that the sum of angle 1 plus angle 8 must equal the sum of angle 5 plus angle 4. By similar reasoning, the sum of angle 2 plus angle 3 must equal the sum of angle 6 plus angle 7.

Suppose now, that the values of angles 2, 3, 6, and 7, after adjustment for the sum of interior angles, areas are as follows:

Table 14-8 – Adjustment for the Sum of Interior Angles.

Angle	Value after First Figure Adjustment		
2	23°	44′	37″
3	42°	19′	08″
Sum	66°	03′	45″
6	39°	37′	47″
7	26°	25′	50″
Sum	66°	03′	37″

The difference between the two sums is 8 seconds. This means that, to make the sums equal, 4 seconds should be subtracted from the 2-3 sum and added to the 6-7 sum. To subtract 4 seconds from the 2-3 sum, you subtract 2 seconds from each angle; to add 4 seconds to the 6-7 sum, you add 2 seconds to each angle.

The final step in quadrilateral adjustment is related to the fact that you can compute the length of a side in a quadrilateral by more than one route. The final step in adjustment is to ensure that, for a given side, you will get the same result, to the desired number of significant figures, regardless of the route your computations take.

This final adjustment is called the log-sine adjustment because it uses the logarithmic sines of the angles. The method is based on the use of side equations to derive an equation from which the sides are eliminated and only the sines of the angles remain. This equation is derived as follows:

Suppose that in *Figure 14-21*, *AB* is the baseline and the length of *CD* is to be computed. By the law of sines,

$$\frac{AD}{\sin 3} = \frac{AB}{\sin 8} \therefore AD = \frac{\sin 3}{\sin 8}$$

By the same law,

$$\frac{CD}{\sin 1} = \frac{AD}{\sin 6} \therefore CD = AD\frac{\sin 1}{\sin 6}$$

Substituting the value of *AD* we have

$$CD = AB\frac{\sin 3 \sin 1}{\sin 8 \sin 6}$$

Again by the law of sines we have

$$\frac{CD}{\sin 4} = \frac{BC}{\sin 7} \therefore CD = BC\frac{\sin 4}{\sin 7}$$

By the same law,

$$\frac{BC}{\sin 2} = \frac{AB}{\sin 5} \therefore BC = AB\frac{\sin 2}{\sin 5}$$

Substituting this value for *BC*, we have

$$CD = AB\frac{\sin 2 \sin 4}{\sin 5 \sin 7}$$

We now have two values for *CD*, as follows:

$$CD = AB\frac{\sin 1 \sin 3}{\sin 6 \sin 8}$$

$$CD = AB\frac{\sin 2 \sin 4}{\sin 5 \sin 7}$$

It follows that

$$AB\frac{\sin 1 \sin 3}{\sin 6 \sin 8} = AB\frac{\sin 2 \sin 4}{\sin 5 \sin 7}$$

Canceling out *AB* we have

$$\frac{\sin 1 \sin 3}{\sin 6 \sin 8} = \frac{\sin 2 \sin 4}{\sin 5 \sin 7}$$

By the law of proportions, this can be expressed as

$$\frac{\sin 1 \sin 3 \sin 5 \sin 7}{\sin 2 \sin 4 \sin 6 \sin 8} = 1$$

You know that in logarithms, instead of multiplying you just add logarithms; also, instead of dividing one number by another, you just subtract the logarithm of the second from the logarithm of the first. Note that the logarithm of 1 is 0.000000. Therefore, the above equation can be expressed as follows:

(log sin 1 + log sin 3 + log sin 5 + log sin 7) – (log sin 2 + log sin 4 + log sin 6 + log sin 8) = 0

Suppose now that after the second figure adjustment, the values of the angles shown in *Figure 14-21* are as follows:

Table 14-9 – Values of Angles after Second Figure Adjustment.

Angle	Value after 2nd Figure Adjustment			Angle	Value after 2nd Figure Adjustment		
1	38°	44′	06″	2	23°	44′	35″
3	42°	19′	06″	4	44°	51′	59″
5	69°	04′	20″	6	39°	37′	49″
7	26°	25′	52″	8	75°	12′	13″

A table of logarithmic functions shows the log sines of these angles to be as follows:

Table 14-10 – Log Sines of Angles.

Angle	Log Sine	Angle	Log Sine
1	9.796380 – 10	2	9.604912 – 10
3	9.828176 – 10	4	9.848470 – 10
5	9.970361 – 10	6	9.804706 – 10
7	9.678478 – 10	8	9.985354 – 10
Sum	9.243395 – 10	Sum	9.243442 – 10

By subtracting the two sums, you get the following:

$$\begin{array}{l} 9.243442\text{-}10 \\ \underline{-9.243395\text{-}10} \\ 0.0000047 \end{array}$$

Therefore, the difference in the sums of the log sines is 0.000047. Since there are eight angles, this means the average difference for each angle is 0.0000059.

The next question is how to convert this log sine difference per angle into terms of angular measurement. To do this, you first determine, by reference to the table of log functions, the average difference in log sine, per second of arc, for the eight angles involved. This is determined from the D values given in the table. For each of the angles shown in *Figure 14-21*, the D value is as follows:

Table 14-11 – D Value (″)

Angle	D Value (″)
1	2.62
3	2.32
5	0.82
7	4.23
2	4.78
4	2.12
6	2.55
8	0.57
Sum	20.01

The average difference in log sine per 1 second of arc, then, is 20.01/8, or 2.5. The average difference in log sine is 5.9; therefore, the average adjustment for each angle is

5.9 +2.5, or about 2 seconds. The sum of the log sines of angles 2, 4, 6, and 8 is greater than that of angles 1, 3, 5, and 7. Therefore, you add 2 seconds each to angles 1, 3, 5, and 7 and subtract 2 seconds each from angles 2, 4, 6, and 8.

Test your Knowledge (Select the Correct Response)

7. Which statement best describes a primary triangulation station?

   A. It is a sighted station only
   B. It is an instrument station only
   C. It is both a sighted station and an instrument station
   D. None of the above

3.5.0 Checking for Precision

Early in this chapter the fact was stated that the precision of a triangulation survey may be classified according to (1) the average triangle closure and (2) the discrepancy between the measured length of a base line and its length as computed through the system from an adjacent base line.

3.5.1 Average Triangle Closure

The check for average triangle closure is made after the station adjustment. Suppose that, for the quadrilateral shown in *Figure 14-21*, the values of the angles in the quadrilateral after station adjustment were as follows:

Table 14-12 – Values after Station Adjustment.

Angle	Value after Station Adjustment
1	38° 44´ 06´´
2	23° 44´ 38´´
3	42° 19´ 09´´
4	44° 52´ 01´´
5	69° 04´ 21´´
6	39° 37´ 48´´
7	26° 25´ 51´´
8	75° 12´ 14´´

The sum of the angles that make up each of the overlapping triangles within the quadrilateral is as follows:

Table 14-13 – Sum of the Angles for Overlapping Triangles.

Triangles	Angles		Triangles	Angles	
ABC	2	23° 44´ 38´´	ABD	1	38° 44´ 06´´
	3	42° 19´ 09´´		2	23° 44´ 36´´
	4	44° 52´ 01´´		8	75° 12´ 14´´
	5	69° 04´ 21´´		3	42° 19´ 09´´
Sum	180° 00´ 09´´		Sum	180° 00´ 07´´	
ACD	1	38° 44´ 06´´	DBC	7	26° 25´ 51´´
	6	39° 37´ 48´´		4	44° 52´ 01´´
	7	26° 25´ 51´´		6	39° 37´ 48´´
	8	75° 12´ 14´´		5	69° 04´ 21´´
Sum	179° 59´ 59´´		Sum	180° 00´ 01´´	

The sum of the closing errors for the four triangles is (09 + 01 + 07 + 01), or 18 seconds. The average triangle closure for the four triangles, then, is 18/4, or 04.5 seconds. For third-order triangulation, the maximum average triangle closure is 05 seconds; therefore, for the third-order work this closure would be acceptable.

3.5.2 Base Line Discrepancy

If *AD* is the base line in *Figure 14-21*, then *BC* would be the adjacent baseline. Let us assume that the baseline *AD* measures 700.00 feet and compute the length of *BC* on the basis of the angles we have adjusted. These angles now measure as follows:

Table 14-14 – Value after Final Adjustment.

Angle	Value after Final Adjustment	Angle	Value after Final Adjustment
1	38° 44´ 08´´	2	23° 44´ 33´´
3	42° 19´ 08´´	4	44° 51´ 57´´
5	69° 04´ 22´´	6	39° 37´ 47´´
7	26° 25´ 54´´	8	75° 12´ 14´´

The natural sine of each of these angles is as follows:

Table 14-15 – Natural Sine of Angles.

Angle	Sine	Angle	Sine
1	0.625727	2	0.402627
3	0.673257	4	0.705448
5	0.934035	6	0.637801
7	0.445130	8	0.966837

You can compute the length of *BC* by first solving triangle *ABD* for *AB* and triangle *ABC* for *BC* and then solving triangle *ACD* for *DC* and triangle *DBC* for *BC.* Using the law of sines and solving triangle *ABD* for side *AB,* we have

$$AB = AD\frac{\sin 8}{\sin 3}$$

$$= AD\frac{\sin 75^{\circ}12'11''}{\sin 42^{\circ}19'08''}$$

$$= 1{,}005.243\ ft$$

Solving triangle ABC for side BC, we have

$$BC = AB\frac{\sin 2}{\sin 5}$$

$$= AD\frac{\sin 23^{\circ}44'33''}{\sin 69^{\circ}04'22''}$$

$$1{,}005.243\frac{(0.402627)}{(0.934035)}$$

$$= 433.322\ ft$$

Solving triangle *ACD* for side CD, we have

$$CD = AD \frac{\sin 1}{\sin 6}$$

$$= AD \frac{\sin 38°44'08''}{\sin 39°37'47''}$$

$$700.00 \frac{(0.625727)}{(0.637824)}$$

$$= 686.724\, ft$$

Solving triangle *DBC* for side *BC*, we have

$$BC = CD \frac{\sin 7}{\sin 4}$$

$$= CD \frac{\sin 26°25'54''}{\sin 44°51'57''}$$

$$686.724 \frac{(0.445130)}{(0.7.5449)}$$

$$= 433.315\, ft$$

Thus we have, by computation of two routes, values for *BC* of 433.322 feet and 433.315 feet. There is a discrepancy here of 0.007 feet. For third-order work this would usually be considered within tolerable limits, and the computed value of *BC* would be taken to be the average between the two, or (to the nearest 0.01 foot) 433.32 feet.

Now suppose that the precision requirements for the base line check are 1/5,000. This means that the ratio between the difference in lengths of the measured and computed base line must not exceed 1/5,000. You measure the base line *BC* and discover that it measures 433.25 feet. For a ratio of error of 1/5,000, the maximum allowable error (discrepancy between computed and measured value of base line) is 433.25/5,000, or 0.08 feet. The error here is (433.32 – 433.25), or 0.07 foot, which is within the allowable limit.

3.6.0 Locations of Points

The end result desired in a triangulation survey is the horizontal locations of the points in the system, by bearing and distance. Methods of converting deflection angles to bearings and converting bearings to exterior or interior angles are described in the EA Basic, Chapter 18.

3.6.1 Bearing and Distance

Figure 14-22 shows the quadrilateral we have been working on, with the computed values of the sides inscribed. Take station *D* as the starting point. Suppose that, by an appropriate method, you have determined the bearing of *DA* to be N15°00´00´´E, as shown. To have a good picture of how you proceed to compute for the bearing of the next line, *AB,* you must superimpose the meridian line through the starting point, laying off approximately the known bearing, in this case, N15°00´00´´E. Now draw your meridian through point *A.*

From *Figure 14-22* you can see that the line *AB* bears southeast, and you can find its bearing by subtracting 15°00´00´´ from angle *A.* Angle *A* is the sum of angles 1 and 2 (38°44´08´´ + 23°44´35´´), or 62°28´43´´, as you should recall from *Figure 14-21*. The bearing angle of *AB,* then, is 62°28´43´´ – 15°00´00´´, or 47°28´43´´. Therefore, the complete bearing of line *AB* is S47°28´43´´E.

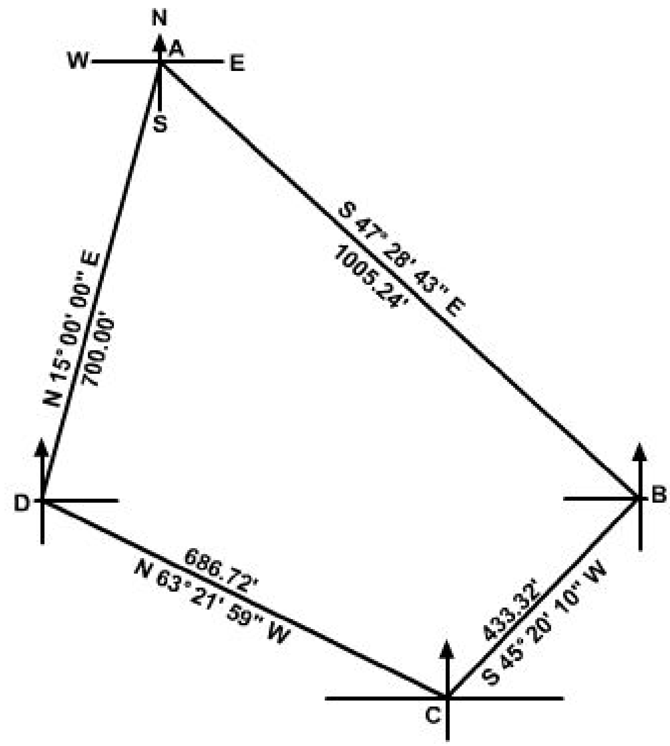


Figure 14-22 — Bearing and distances of a quadrilateral.

You would find the bearing of *BC* and *CD* similarly, except that you have to watch for the angle you are after. Always remember that a bearing angle does not exceed 90° and is always reckoned from north or south. To find the bearing of *BC,* you must find the sum of angle *B* (angles 3 and 4, *Figure 14-21*) plus the bearing angle of *AB*, and then subtract it from 180°; you can see that *BC* bears southwest, so just add this designation to the proper place in the bearing angle for *BC.* In this case, the bearing of *BC* will be 180°00´00´´ – (42°19´08´´ + 44°51´59´´ + 47°28´43´´), or S45°20´10´´W. The bearing of *CD* is equal to angle *C* minus the bearing angle of *BC.*

3.6.2 Coordinates

Suppose that you are tying the quadrilateral shown in *Figure 14-22* into a state grid system. The nearest monument in this system lies 1,153.54 feet from station *D,* bearing S50°16´36´´W from *D,* as shown in *Figure 14-23*. This means that the bearing from the monument to *D* is N50°16´36´´E. Suppose that the grid coordinates of the monument are y = 373,462.27 feet and x = 562,496.37 feet.

The latitude of the line from the monument to station D is 1,153.54 cos 50°16´36´´, or 737.21 feet.

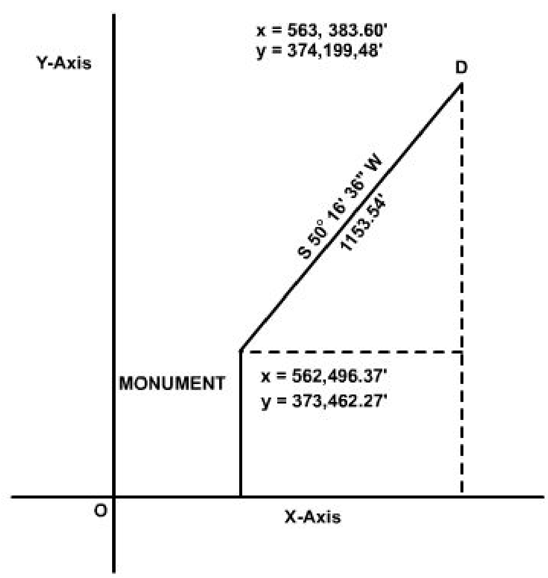


Figure 14-23─Coordinates.

The departure of the same line is 1,15354 sin 50°16´36´´, or 887.23 feet. The y coordinate of station *D* equals the y coordinate of the monument plus the latitude of the line from the monument to *D,* or 373,462.27 + 737.21, or 374,199.48 feet. The x coordinate of station *D* equals the x coordinate of the monument plus the departure of the line from the monument to *D,* or 562,496.37 + 887.23, or 563,383.60 feet.

Knowing the coordinates of station *D,* you can now determine the coordinates of station *A.* The latitude of *DA* is 700.00 cos 15°00´00´´, or 676.15 feet. The departure of *DA* is 700.00 sin 15°00´00´´, or 181.17 feet. The y coordinate of station *A* is equal to the y coordinate of station *D* plus the latitude of *DA,* or 374,199.48 + 676.15, or 674,875.63 feet. The x coordinate of station *A* is equal to the x coordinate of station *D* plus the departure of *DA,* o r 563,383.60 + 181.17, or 563,564.77 feet. The coordinates of the other stations can be similarly determined.

Test your Knowledge (Select the Correct Response)

8. **(True or False)** The end result desired in a triangulation survey is the horizontal locations of the points in the system, by bearing and distance.

   A. True
   B. False

Summary

This chapter discussed the important aspects of carrying out your duties as a Triangulation Survey Party Chief including understanding time designations and using triangulation methods essential to accurate survey results.

In the section on triangulation you received information on the purpose and kinds of triangulation networks, the steps involved in a triangulation survey, and the computations involved in establishing horizontal control points using triangulation.

Finally, the chapter also addressed satellite survey systems and how they are used in locating point positions on the surface of the earth through observation of data received from Global Positioning Satellites (GPS).

Trade Terms Introduced in this Chapter

Local Civil Time	United States terminology from 1925 to 1952 for local mean time.
Standard Time Meridian	Every meridian east or west of Greenwich that is numbered 15° or a multiple of 15° (such as 30° east or west, 45° east or west, 60° east or west, and so on) is designated as a standard time meridian.
Triangulation	The most common type of geodetic survey; it requires that distances be measured only at the beginning, at specified intervals, and at the end of the survey.
Chain of Polygons	A system in which a number of adjacent triangles are combined to form a polygon.
Geodetic	A branch of earth sciences; the scientific discipline that deals with the measurement and representation of the earth.
Triangulation Station	A fixed surveying station for the geodetic surveying and other surveying projects on nearby areas.
Chain of Quadrilaterals	A quadrilateral is technically a polygon, and a chain of quadrilaterals would be technically a chain of polygons.
Intervisibility	A relative, localized pattern of limitations on observation, caused by variations in terrain elevation.
Order of Precision	An order of precision used for land surveying; varies directly with the value of the land and also with other circumstances.
Least Squares Method	The most rigorous and accurate of adjustment methods; involves the computation of the most probable values of the adjusted quantities.