



Online Continuing Education for Professional Engineers
Since 2009

Advanced Electrical Theory

PDH Credits:

4 PDH

Course No.:

AET101

Publication Source:

US Navy (NAVEDTRA)

Engineering Aid Basics

"Chapter 1 – Advanced Electrical Theory"

Pub. # NAVEDTRA 14027A

DISCLAIMER:

All course materials available on this website are not to be construed as a representation or warranty on the part of Online-PDH, or other persons and/or organizations named herein. All course literature is for reference purposes only, and should not be used as a substitute for competent, professional engineering council. Use or application of any information herein, should be done so at the discretion of a licensed professional engineer in that given field of expertise. Any person(s) making use of this information, herein, does so at their own risk and assumes any and all liabilities arising therefrom.

Chapter 1

Advanced Electrical Theory

Topics

- 1.0.0 AC Generation
- 2.0.0 Inductive Circuit
- 3.0.0 Capacitive Circuit
- 4.0.0 Reactance and Impedance
- 5.0.0 LC and RLC Circuits
- 6.0.0 Power in AC Circuits
- 7.0.0 Power Factor

To hear audio, click on the box.



Overview

Your knowledge, understanding, and application of advanced electrical theory are very important for the safe conduct and completion of your job as a Construction Electrician. Upon completion of this chapter you should have an understanding of electrical theory in regards to alternating current circuits and power within those circuits. As a Construction Electrician, you need the knowledge of the concepts and principles when dealing with alternating and direct current as you apply electrical and electronic theory in your everyday duties.

Objectives

When you have completed this chapter, you will be able to do the following:

1. Understand Alternating Current (AC) generation.
2. Understand the principles of an inductive circuit.
3. Understand the principles of a capacitive circuit.
4. Understand the principles of a resistive circuit.
5. Understand the principles of inductive capacitor circuit (LC) and resistor inductor capacitor circuit (RLC).
6. Understand the power in alternating current (AC) circuits.
7. Understand the meaning of power factor.

1.0.0 ALTERNATING CURRENT (AC) GENERATION

A current-carrying conductor produces a magnetic field around itself. When a conductor is in a magnetic field and either the field or the conductor moves, an electromagnetic field (emf) (voltage) is induced in the conductor. This effect is called electromagnetic induction. *Figures 1-1 and 1-2* show a suspended loop of wire (conductor) being rotated (moved) in a clockwise direction through the magnetic field between the poles of a permanent magnet. For ease of explanation, the loop has been divided into a dark half and light half. Notice in (A) of *Figure 1-1, View A* that the dark half is moving along (is

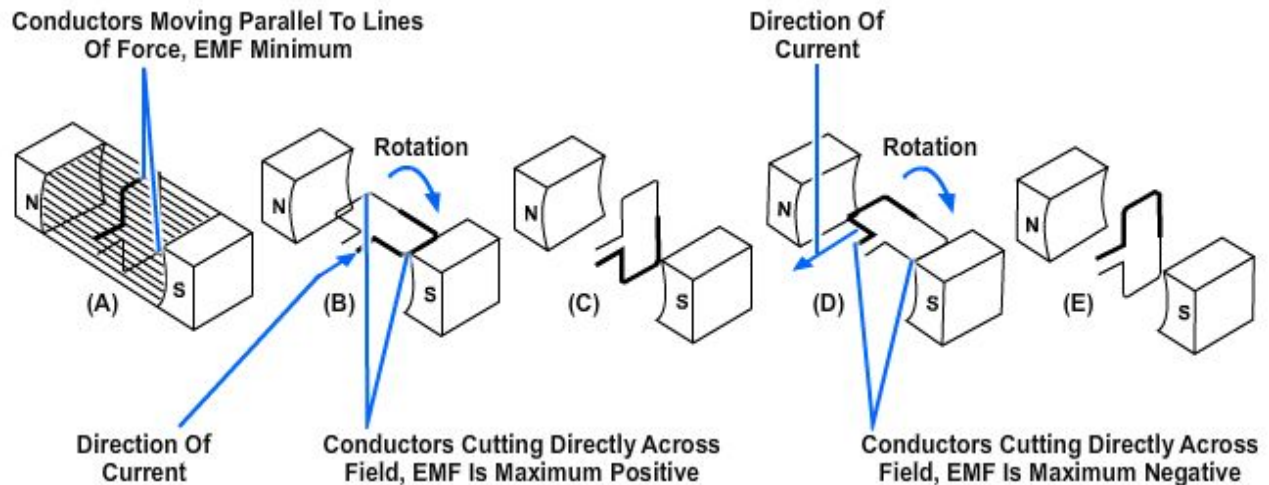


Figure 1-1 – Simple alternating-current generator.

parallel to) the lines of force. Consequently, it is cutting NO lines of force. The same is true of the light half, which is moving in the opposite direction. Since the conductors are cutting no lines of force, no emf is induced. As the loop rotates toward the position shown in (B), it cuts more and more lines of force per second which induces an ever-increasing voltage, because it is cutting more directly across the field (lines of force). At (B), the conductor is shown completing one-quarter of a complete revolution, or 90° , of a complete circle. Because the conductor is now cutting directly across the field, the voltage induced in the conductor is maximum. When the value of induced voltage at various points during the rotation from (A) to (B) is plotted on a graph and the points connected, a curve appears (*Figure 1-2*).

As the loop continues to be rotated toward the position shown in (C) of *Figure 1-2*, it cuts fewer and fewer lines of force. The induced voltage decreases from its peak value. Eventually, the loop is once again moving in a plane parallel to the magnetic field, and no emf is induced in the conductor. The loop has now been rotated through half a circle, one alternation or 180° . If the preceding quarter-cycle is plotted, it appears as shown in *Figure 1-2*.

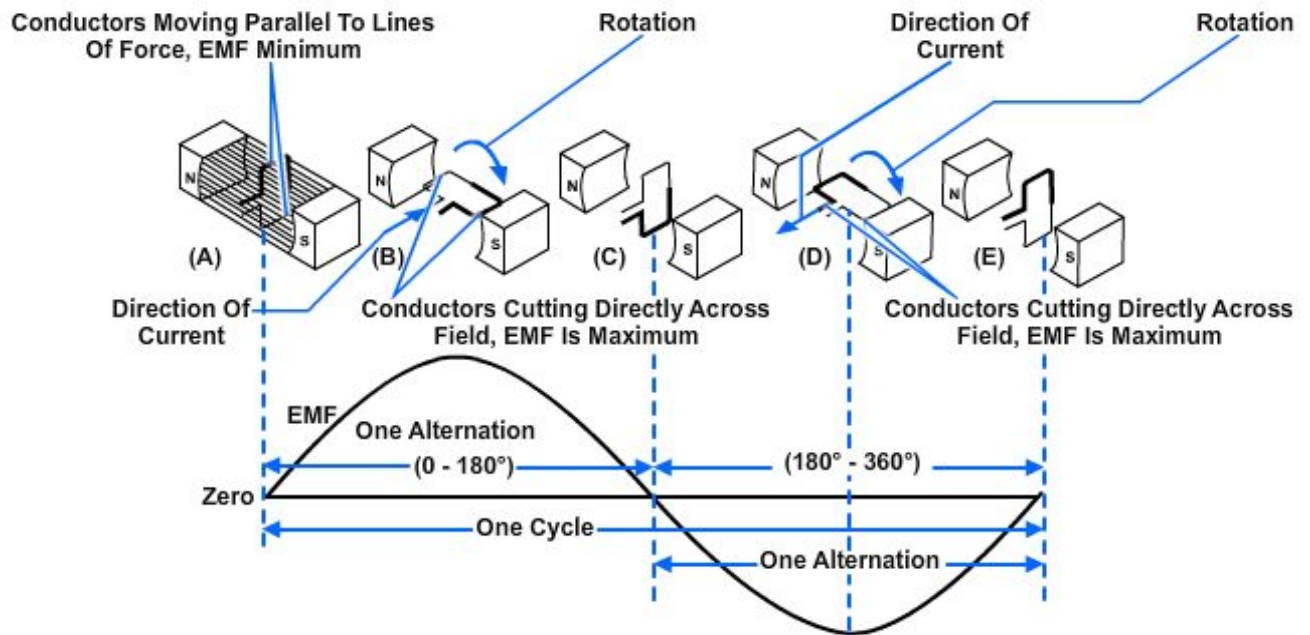


Figure 1-2 — Basic alternating-current generator.

When the same procedure is applied to the second half of rotation (180° through 360°), the curve appears (*Figure 1-2*). Notice the only difference is in the polarity of the induced voltage. Where previously the polarity was positive, it is now negative. The **sine curve** shows the value of induced voltage at each instant of time during rotation of the loop. Notice that this curve contains 360°, or two alternations. Two alternations represent one complete cycle of rotation.

Assuming a closed path is provided across the ends of the conductor loop, you can determine the direction of current in the loop by using the **left-hand rule** for generators (*Figure 1-3*). The left-hand rule is applied as follows:

- Place your left hand on *Figure 1-3* with the fingers in the direction shown in the illustration. Your thumb will now point in the direction of rotation (relative movement of the wire to the magnetic field)
- Your forefinger will point in the direction of magnetic flux (north to south)
- Your middle finger (pointing back toward you) will point in the direction of electron current flow.

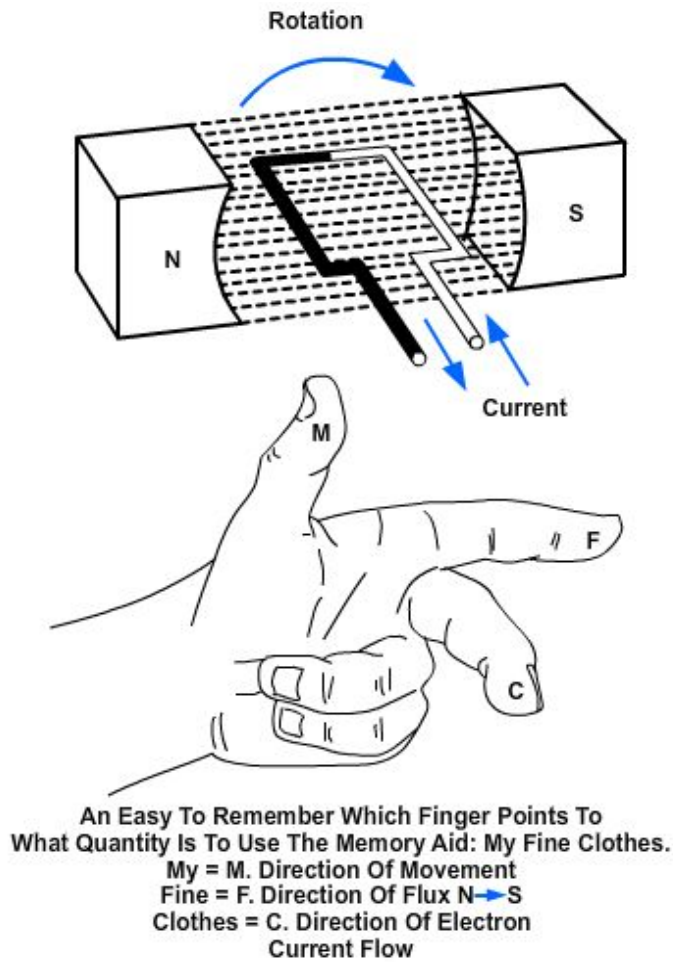


Figure 1-3 – Left-hand rule for generators.

By applying the left-hand rule to the dark half of the loop in (B) of *Figure 1-2*, you will find that the current flows in the direction indicated by the heavy arrow. Similarly, by using the left-hand rule on the light half of the loop, you will find that the current flows in the opposite direction. The two induced voltages in the loop add together to form one total emf. It is this emf which causes the current in the loop.

When the loop rotates to the position shown in (D) of *Figure 1-2*, the action reverses. The dark half is moving up instead of down, and the light half is moving down instead of up. By applying the left-hand rule once again, you will see that the total induced emf and its resulting current have reversed direction. The voltage builds up to maximum in this new direction, as shown by the sine curve in *Figure 1-2*. The loop finally returns to its original position (E), at which point voltage is again zero. The sine curve represents one complete cycle of voltage generated by the rotating loop. All of the figures in this section show the wire loop moving in a clockwise direction. In actual practice, the loop can be moved clockwise or counterclockwise. Regardless of the direction of movement, the left-hand rule applies.

If the loop is rotated through 360° at a steady rate, and if the strength of the magnetic field is uniform, the voltage produced is a sine wave of voltage, as indicated in *Figure 1-3*. Continuous rotation of the loop will produce a series of sine-wave voltage cycles or, in other words, an ac voltage.

1.1.0 Sine Wave

An individual cycle of any sine wave represents a definite amount of time. Notice that *Figure 1-4* shows 2 cycles of a sine wave which has a frequency of 2 hertz (Hz). Since two cycles occur each second, one cycle must require one-half second of time. The time required to complete one cycle of a waveform is called the period of the wave. In *Figure 1-4*, the period is one-half second. The relationship between time (t) and frequency (f) is indicated by the following formulas:

$$t = \frac{1}{f} \text{ and } f = \frac{1}{t}$$

where t = period in seconds and

f = frequency in hertz

Each cycle of the sine wave shown in *Figure 1-4* consists to two identically shaped variations in voltage. The variation which occurs during the time the voltage is positive is called the positive alternation. The variation which occurs during the time the voltage is negative is called the negative alternation. In a sine wave, the two alternations are identical in size and shape, but opposite in polarity.

The distance from zero to the maximum value of each alternation is called the amplitude. The amplitude of the positive alternation and the amplitude of the negative alternation are the same.

The time it takes for a sine wave to complete one cycle is defined as the period of the waveform. The distance traveled by the sine wave during this period is referred to as wavelength. Wavelength, indicated by the symbol λ (Greek lambda), is the distance along the waveform from one point to the same point on the next cycle. You can observe this relationship by examining *Figure 1-5*. The point on the waveform where measurement of the wavelength begins is not important as long as the distance is measured to the same point on the next cycle (*Figure 1-6*).

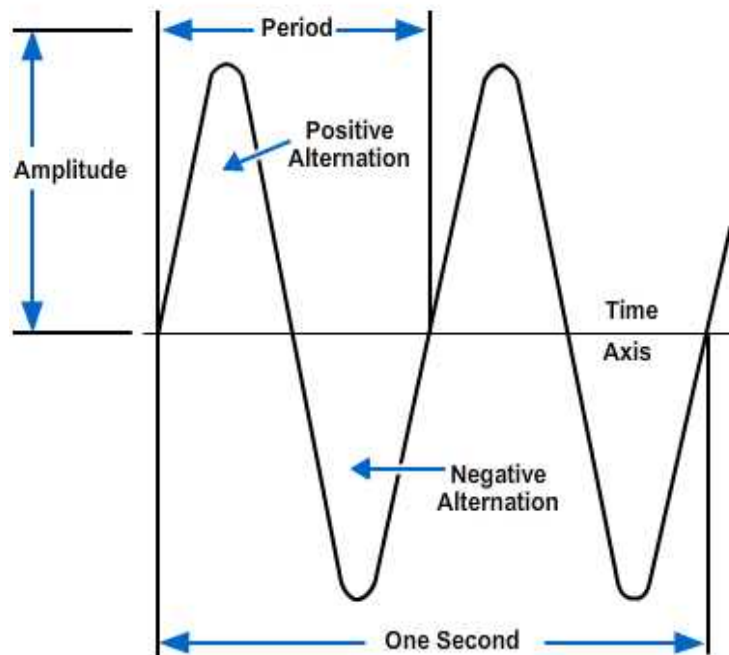


Figure 1-4 — Period of a sine wave.

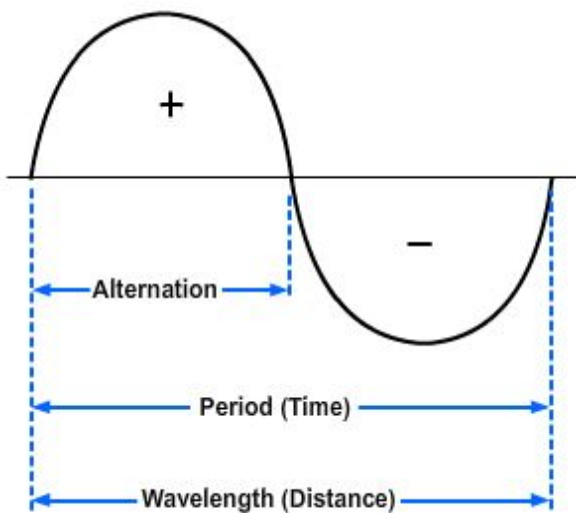


Figure 1-5 – Wavelength.

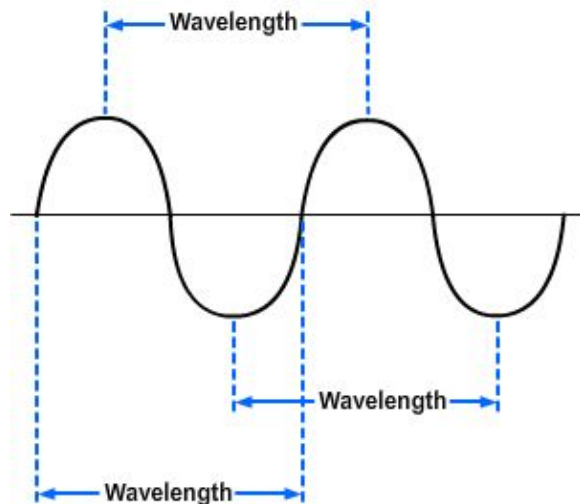


Figure 1-6 – Wavelength measurement.

1.2.0 Frequency

If the loop in the *Figure 1-2 (A)* makes one complete revolution each second, the generator produces one complete cycle of ac during each second (1 Hz). Increasing the number of revolutions to two per second will produce two complete cycles of ac per second (2 Hz). The number of complete cycles of alternating current or voltage completed each second is referred to as the frequency. Frequency is always measured and expressed in hertz.

Alternating-current frequency is an important term to understand since most ac electrical equipments require a specific frequency for proper operation.

1.3.0 Peak Value

Figure 1-7 shows the positive alternation of a sine wave (a half-cycle of ac) and a dc waveform that occur simultaneously. Note that the dc starts and stops at the same moment as does the positive alternation, and that both waveforms rise to the same maximum value. However, the dc values are greater than the corresponding ac values at all points except the point at which the positive alternation passes through its maximum value. At this point the dc and ac values are equal. This point on the sine wave is referred to as the maximum or peak value.

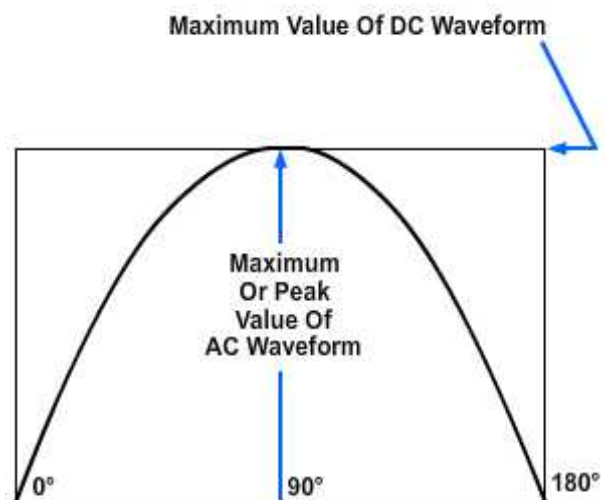


Figure 1-7 – Maximum or peak value.

During each complete cycle of ac there are always two maximum or peak values, one for the positive half-cycle and the other for the negative half-cycle. The difference

between the peak positive value and the peak negative value is called the peak-to-peak value of the sine wave. This value is twice the maximum or peak value of the sine wave and is sometimes used for measurement of ac voltages. Note the difference between peak and peak-to-peak values (*Figure 1-8*). Usually alternating voltage and current are expressed in effective values, which will be discussed later, rather than in peak-to-peak values.

1.3.1 Instantaneous Value

The instantaneous value of an alternating voltage or current is the value of voltage or current at one particular instant. The value may be zero if the particular instant is the time in the cycle at which the polarity of the voltage is changing. It may also be the same as the peak value if the selected instant is the time in the cycle at which the voltage or current stops increasing and starts decreasing. There are actually an infinite number of instantaneous values between zero and the peak value.

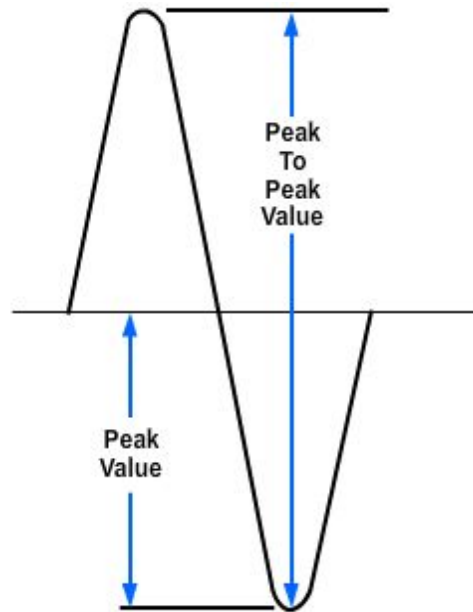


Figure 1-8 – Peak and peak-to-peak values.

1.4.0 Effective Value

E_{max} , E_{avg} , I_{max} , and I_{avg} are values used in ac measurements. Another value used is the effective value of ac. This is the value of alternating voltage or current that will have the same effect on a resistance as a comparable value of direct voltage or current will have on the same resistance.

Recall that when current flows in a resistance, heat is produced. When direct current flows in a resistance, the amount of electrical power converted into heat equals I^2R watts. However, since an alternating current having a maximum value of one ampere does not maintain a constant value, the alternating current will not produce as much heat in the resistance as will a direct current of one ampere.

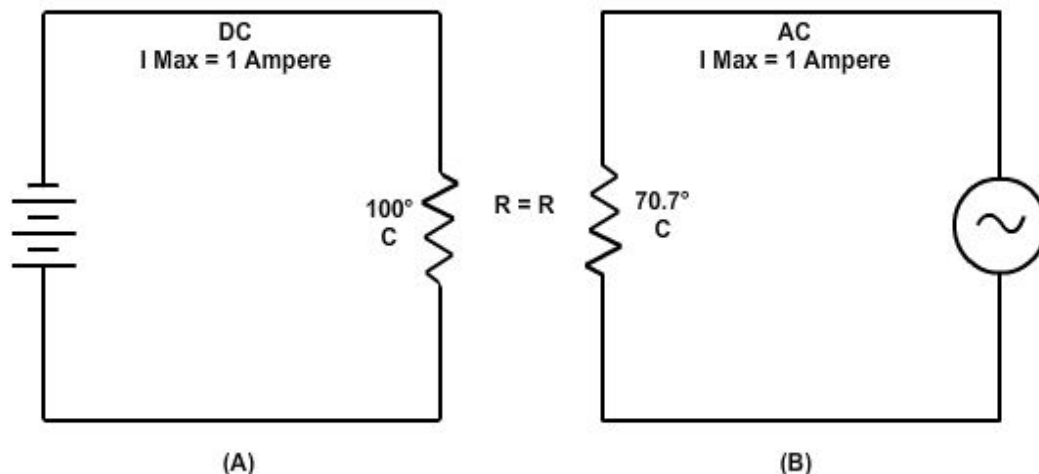


Figure 1-9 – Heating effect of ac and dc.

Figure 1-9 compares the heating effect of one ampere of dc to the heating effect of one ampere of ac. Examine Views A and B of Figure 1-9 and notice that the heat (70.7° C) produced by one ampere of alternating current (an ac with a maximum value of one ampere) is only 70.7 percent of the heat (100° C) produced by one ampere of direct current. Mathematically that is:

$$\frac{\begin{array}{l} \text{The heating effect} \\ \text{of 1 maximum} \\ \text{a.c. ampere} \end{array}}{\begin{array}{l} \text{The heating effect} \\ \text{of 1 maximum} \\ \text{d.c. ampere} \end{array}} = \frac{70.7^{\circ} \text{ C}}{100^{\circ} \text{ C}} = 0.707$$

Therefore, for effective value of ac ($I_{\text{eff}} = 0.707 \times I_{\text{max}}$).

The rate at which heat is produced in a resistance forms a convenient basis for establishing an effective value of alternating current, and is known as the “heating effect” method. An alternating current is said to have an effective value of one ampere when it produces heat in a given resistance at the same rate as does one ampere of direct current.

You can compute the effective value of a sine wave of current to a fair degree of accuracy by taking equally-spaced instantaneous values of current along the curve and extracting the square root of the average sum of the squared values.

For this reason, the effective value is often called the “root-mean-square” (rms) value. Thus,

$$I_{\text{eff}} = \sqrt{\text{Average of the sum of the squares of } I_{\text{inst}}}$$

Stated another way, the effective or rms value (I_{eff}) of a sine wave of current is 0.707 times the maximum value of current (I_{max}). Thus, $I_{\text{eff}} = 0.707 \times I_{\text{max}}$. When I_{eff} is known, you can find I_{max} by using the formula $I_{\text{max}} = 1.414 \times I_{\text{eff}}$. You might wonder where the constant 1.414 comes from. To find out, examine Figure 1-9 again and read the following explanation. Assume that the dc in Figure 1-9 (A) is maintained at one ampere and the resistor temperature at 100° C. Also assume that the ac in Figure 1-9 (B) is increased until the temperature of the resistor is 100° C. At this point it is found that a maximum ac value of 1.414 amperes is required in order to have the same heating effect as direct current. Therefore, in the ac circuit the maximum current required is 1.414 times the effective current. It is important for you to remember the above relationship and that the effective value (I_{eff}) of any sine wave of current is always 0.707 times the maximum value (I_{max}).

Since alternating current is caused by an alternating voltage, the ratio of the effective value of voltage to the maximum value of voltage is the same as the ratio of the effective value of current to the maximum value of current. Stated another way, the effective or rms value (E_{eff}) of a sine wave of voltage is 0.707 times the maximum value of voltage (E_{max}). Thus,

$$E_{\text{eff}} = \sqrt{\text{Average of the sum of the squares of } E_{\text{inst}}}$$

or,

$$E_{\text{eff}} = 0.707 \times E_{\text{max}}$$

and,

$$E_{\max} = 1.414 \times E_{\text{eff}}$$

When an alternating current or voltage value is specified in a book or on a diagram, the value is an effective value unless there is a definite statement to the contrary. Remember that all meters, unless marked to the contrary, are calibrated to indicate effective values of current and voltage.

1.5.0 Average Value

The average value of an alternating current or voltage is the average of all the instantaneous values during one alternation. Since the voltage increases from zero to peak value and decreases back to zero during one alternation, the average value must be some value between those two limits. You could determine the average value by adding together a series of instantaneous values of the alternation (between 0° and 180°), and then dividing the sum by the number of instantaneous values used. The computation would show that one alternation of a sine wave has an average value equal to 0.636 times the peak value. The formula for average voltage is $E_{\text{avg}} = 0.636 \times E_{\max}$ where E_{avg} is the average voltage of one alternation, and E_{\max} is the maximum or peak voltage. Similarly, the formula for average current is $I_{\text{avg}} = 0.636 \times I_{\max}$ where I_{avg} is the average current in one alternation, and I_{\max} is the maximum or peak current.

Do not confuse the above definition of an average value with that of the average value of a complete cycle. Because the voltage is positive during one alternation and negative during the other alternation, the average value of the voltage values occurring during the complete cycle is zero.

2.0.0 INDUCTIVE CIRCUIT

The study of inductance presents a very challenging but rewarding segment of electricity. It is challenging in the sense that, at first, it will seem that new concepts are being introduced. You will realize as this chapter progresses that these “new concepts” are merely extensions and enlargements of fundamental principles that were previously introduced in CE Basic. The study of inductance is rewarding in the sense that a thorough understanding of it will enable you to acquire a working knowledge of electrical circuits more rapidly.

2.1.0 Inductance

Inductance is the characteristic of an electrical circuit that opposes the starting, stopping, or a change in value of current. The symbol for inductance is L and the basic unit of inductance is the Henry (H). One henry is equal to the inductance required to induce one volt in an inductor by a change of current of one ampere per second.

You do not have to look far to find a physical analogy of inductance. Anyone who has ever had to push a heavy load (wheelbarrow, car, etc.) is aware that it takes more work to start the load moving than it does to keep it moving. Once the load is moving, it is easier to keep the load moving than to stop it again. This is because the load possesses the property of *inertia*. Inertia is the characteristic of mass which opposes a change in velocity. Inductance has the same effect on current in an electrical circuit as inertia has on the movement of a mechanical object. It requires more energy to start or stop current than it does to keep it flowing.

2.2.0 Inductive Reactance

When the current flowing through an inductor continuously reverses itself, as in the case of an ac source, the inertia effect of the counter electromotive force (cemf) is greater than with dc. The greater the amount of inductance (L), the greater the opposition from this inertia effect. Also, the faster the reversal of current, the greater this inertial opposition. This opposing force which an inductor presents to the flow of alternating current cannot be called resistance, since it is not the result of friction within a conductor. The name given to it is inductive reactance because it is the “reaction” of the inductor to the changing value of alternating current. Inductive reactance is measured in ohms and its symbol is X_L .

As you learned already in this chapter, the induced voltage in a conductor is proportional to the rate at which magnetic lines of force cut the conductor. The greater the rate (the higher the frequency), the greater the cemf. Also, the induced voltage increases with an increase in inductance; the more ampere-turns, the greater the cemf. Reactance, then, increases with an increase of frequency and with an increase of inductance. The formula for inductive reactance is as follows:

$$X_L = 2\pi fL$$

Where:

X_L is inductive reactance in ohms,

2π is a constant in which the Greek letter π ,

called “pi” represents 3.1416 and $2 \times \pi =$
6.28 approximately.

F is frequency of the alternating current in Hz.

L is inductance in henrys.

The following example problem illustrates the computation of X_L .

Given: $f = 60 \text{ Hz}$

$L = 20 \text{ H}$

Solution: $X_L = 2\pi fL$

$X_L = 6.28 \times 60 \text{ Hz} \times 20 \text{ H}$

$X_L = 7,536\Omega$

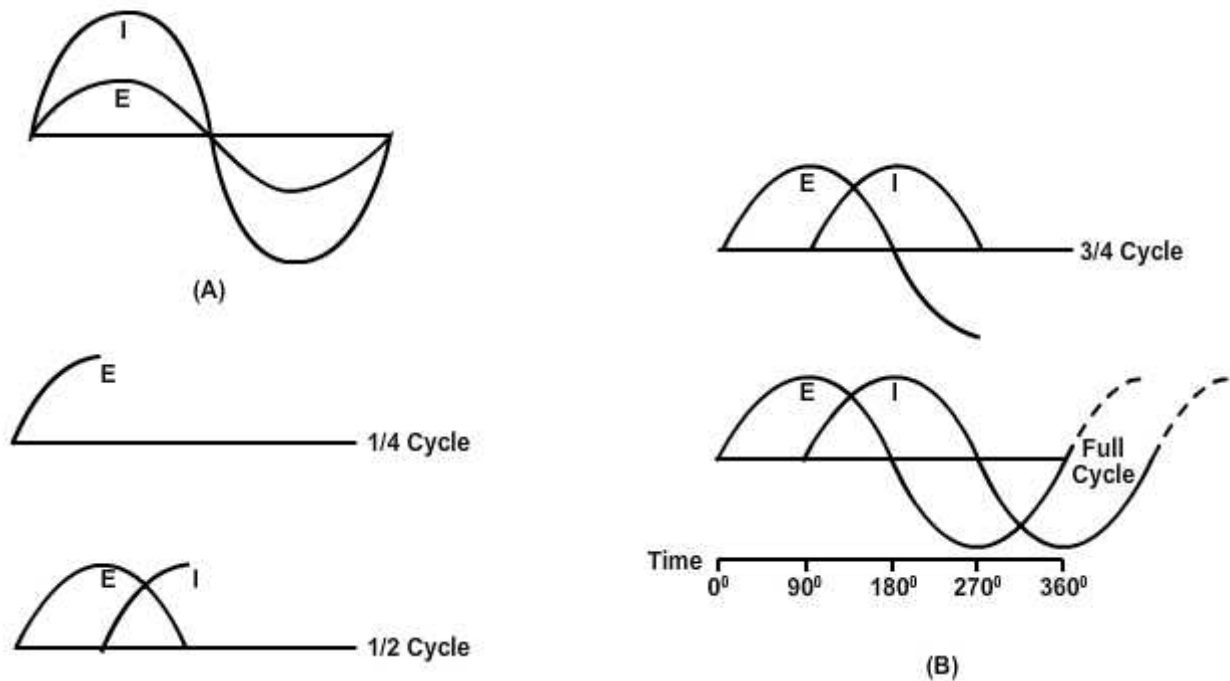


Figure 1-10 – Voltage and current waveform in an inductive circuit.

When two things are in step, going through a cycle together, falling together and rising together, they are in phase. When they are out of phase, the angle of lead or lag – the number of electrical degrees by which one of the values leads or lags the other – is a measure of the amount they are out of step. The time it takes the current in an inductor to build up to maximum and to fall to zero is important for another reason. It helps illustrate a very useful characteristic of inductive circuits - the current through the inductor always lags the voltage across the inductor.

A circuit having pure resistance (if such a thing were possible) would have the alternating current through it and the voltage across it rising and falling together. This is illustrated in *Figure 1-10 (A)*, which shows the sine waves for current and voltage in a purely resistive circuit having an ac source. The current and voltage do not have the same amplitude, but they are in phase.

In the case of a circuit having inductance, the opposing force of the counter emf would be enough to keep the current from remaining in phase with the applied voltage. In a dc circuit containing pure inductance the current takes time to rise to maximum even though the full applied voltage is immediately at maximum. *Figure 1-10 (B)* shows the wave forms for a purely inductive ac circuit in steps of quarter-cycles.

2.3.0 Voltage and Current

With an ac voltage, in the first quarter-cycle (0° to 90°) the applied voltage is continually increasing. If there was no inductance in the circuit, the current would also increase during this first quarter-cycle. You know this circuit does have inductance. Since inductance opposes any change in current flow, no current flows during the first quarter-cycle. In the next quarter-cycle (90° to 180°) the voltage decreases back to zero; current begins to flow in the circuit and reaches a maximum value at the same instant the voltage reaches zero. The applied voltage now begins to build up to maximum in the other direction, to be followed by the resulting current. When the voltage again reaches its maximum at the end of the third quarter-cycle (270°) all values are exactly opposite to what they were during the first half-cycle. The applied voltage leads the resulting current by one quarter-cycle or 90° degrees. To complete the full 360° cycle of the voltage, the voltage again decreases to zero and the current builds to a maximum value.

You must not get the idea that any of these values stops cold at a particular instant. Until the applied voltage is removed, both current and voltage are always changing in amplitude and direction.

As you know, the sine wave can be compared to a circle. Just as you mark off a circle into 360 degrees, you can mark off the time of one cycle of a sine wave into 360 electrical degrees. This relationship is shown in *Figure 1-11*. By referring to this figure you can see why the current in a purely inductive circuit is said to lag the voltage by 90 degrees. Furthermore, by referring to *Figures 1-11* and *1-10 (A)* you can see why the current and voltage are said to be in phase in a purely resistive circuit. In a circuit having both resistance and inductance then, as you would expect, the current lags the voltage by an amount somewhere between 0 and 90 degrees.

A simple memory aid to help you remember the relationship of voltage and current in an inductive circuit is the word ELI. Since E is the symbol of voltage, L is the symbol for inductance, and I is the symbol for current, the word ELI demonstrates that current comes after (Lags) voltage in an inductor.

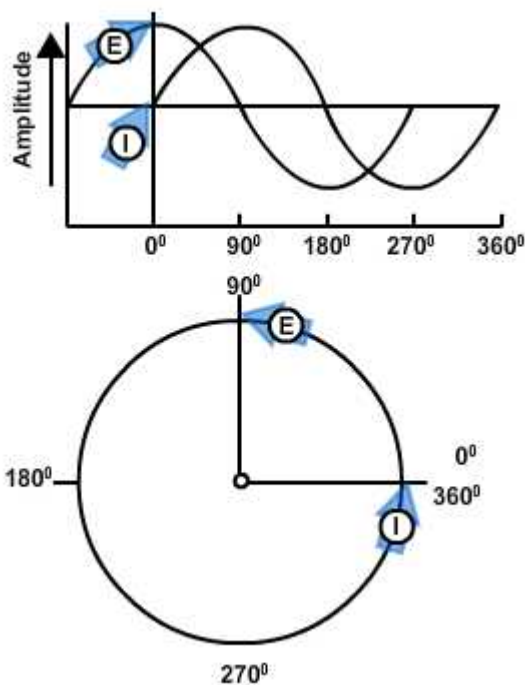


Figure 1-11 — Comparison of sine wave and circle in an inductive circuit.

3.0.0 CAPACITIVE CIRCUIT

3.1.0 Capacitance

Capacitance is the property of a circuit which opposes any change in the circuit voltage. The effect of capacitance may be seen in any circuit where the voltage is changing. Capacitance is usually defined as the ability of a circuit to store electrical energy. This energy is stored in an electrostatic field. The device used in an electrical circuit to store this charge (energy) is called a capacitor. The basic unit of measurement of capacitance is the **farad** (F). A one-farad capacitor will store one coulomb of charge (energy) when a potential of one volt is applied across the capacitor plates. The farad is an enormously large unit of capacitance. More practical units are the microfarad (μF) or the picofarad (pF).

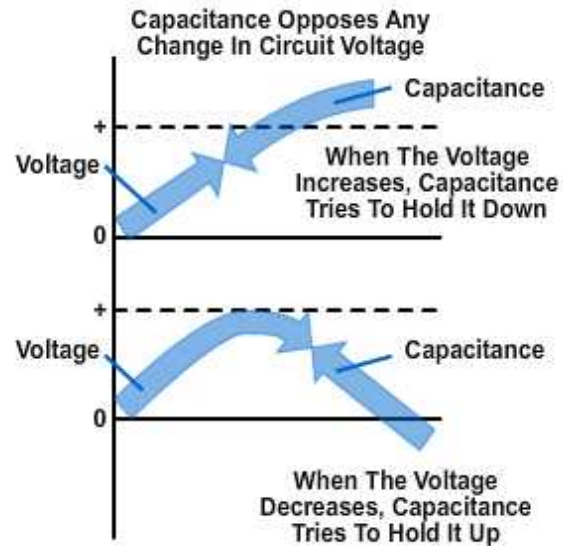


Figure 1-12 – Example of capacitance opposing change in circuit voltage.

3.1.1 Capacitor

A capacitor is a physical device consisting of two pieces of conducting material separated by an insulating material. This insulating material is referred to as the **dielectric**. Because the dielectric is an insulator, no current flows through the capacitor. If the dielectric breaks down and becomes a conductor, the capacitor can no longer hold a charge and is useless. The ability of a dielectric to hold a charge without breaking down is referred to as the dielectric strength. The measure of the ability of the dielectric material to store energy is called the dielectric constant. The dielectric constant is a relative value based on 1.0 for a vacuum.

3.2.0 Capacitive Reactance

So far you have been dealing with the capacitor as a device which passes ac and in which the only opposition to the alternating current has been the normal circuit resistance present in any conductor. However, capacitors themselves offer a very real opposition to current flow. This opposition arises from the fact that, at a given voltage and frequency, the number of electrons which go back and forth from plate to plate is limited by the storage ability – that is, the capacitance of the capacitor. As the capacitance is increased, a greater number of electrons change plates every cycle, and since current is a measure of the number of electrons passing a given point in a given time, the current is increased.

Increasing the frequency will also decrease the opposition offered by a capacitor. This occurs because the number of electrons which the capacitor is capable of handling at a given voltage will change plates more often. As a result, more electrons will pass a given point in a given time, resulting in greater current flow. The opposition which a capacitor offers to ac is therefore inversely proportional to frequency and to capacitance. This opposition is called capacitive reactance. You may say that capacitive reactance decreases with increasing frequency or, for a given frequency, the capacitive

reactance decreases with increasing capacitance. The symbol for capacitive reactance is X_C .

Now you can understand why it is said that the X_C varies inversely with the product of the frequency and capacitance. The formula is:

$$X_C = \frac{1}{2\pi fC}$$

Where:

X_C is capacitive reactance in ohms

F is frequency in hertz

C is capacitance in farads

2π is 6.28 (2 x 3.1416)

The following example problem illustrates the computation of X_C

Given: $f = 100 \text{ Hz}$

$C = 50 \mu\text{F}$

Solution: $X_C = \frac{1}{2\pi fC}$

$$X_C = \frac{1}{6.28 \times 100 \text{ Hz} \times 50 \mu\text{F}}$$

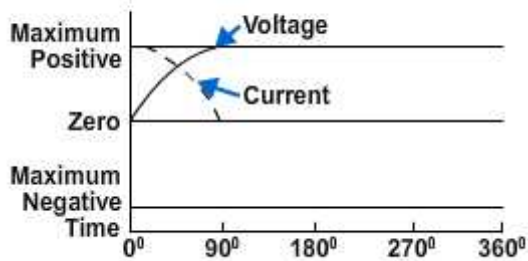
$$X_C = \frac{1}{.0314} \Omega$$

$$X_C = 31.8 \Omega \text{ or } 32 \Omega$$

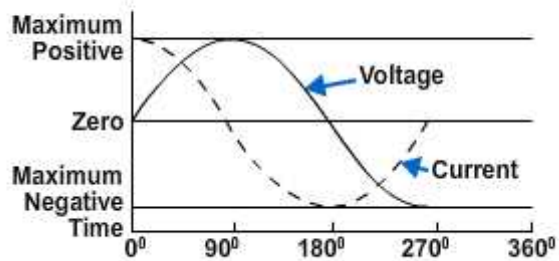
3.3.0 Voltage and Current

The four parts of *Figure 1-13* show the variation of the alternating voltage and current in a capacitive circuit, for each quarter of one cycle. The solid line represents the voltage across the capacitor, and the dotted line represents the current. The line running through the center is the zero, or reference point, for both the voltage and the current. The bottom line marks off the time of the cycle in terms of electrical degrees. Assume that the ac voltage has been acting on the capacitor for some time before the time represented by the starting point of the sine wave in the figure.

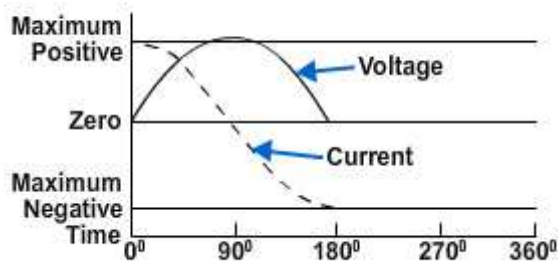
At the beginning of the first quarter-cycle (0° to 90°) the voltage has just passed through zero and is increasing in the positive direction. Since the zero point is the steepest part of the sine wave, the voltage is changing at its greatest rate. The charge on a capacitor varies directly with the voltage, and therefore the charge on the capacitor is also changing at its greatest rate at the beginning of the first quarter-cycle. In other words, the greatest number of electrons are moving off one plate and onto the other plate. Thus, the capacitor current is at its maximum value, as *View A* of *Figure 1-13* illustrates.



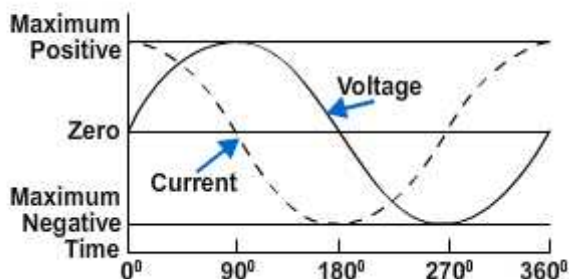
(A)



(C)



(B)



(D)

Figure 1-13 – Phase relationship of voltage and current in a capacitive circuit.

As the voltage proceeds toward maximum at 90 degrees, its rate of change becomes less and less, hence the current must decrease toward zero. At 90 degrees the voltage across the capacitor is maximum, the capacitor is fully charged, and there is no further movement of electrons from plate to plate. That is why the current at 90 degrees is zero.

At the end of this first quarter-cycle the alternating voltage stops increasing in the positive direction and starts to decrease. It is still a positive voltage, but to the capacitor the decrease in voltage means that the plate which has just accumulated an excess of electrons must lose some electrons. The current flow, therefore, must reverse its direction. *Figure 1-13, View B* shows the current curve to be below the zero line (negative current direction) during the second quarter-cycle (90° to 180°).

At 180 degrees the voltage has dropped to zero *Figure 1-13, View C*. This means that for a brief instant the electrons are equally distributed between the two plates; the current is maximum because the rate of change of voltage is maximum. Just after 180 degrees the voltage has reversed polarity and starts building up its maximum negative peak, which is reached at the end of the third quarter-cycle (180° to 270°). During this third quarter-cycle the rate of voltage change gradually decreases as the charge builds to a maximum at 270 degrees. At this point the capacitor is fully charged and it carries the full impressed voltage. Because the capacitor is fully charged there is no further exchange of electrons; therefore, the current flow is zero at this point. The conditions are exactly the same as at the end of the first quarter-cycle (90°) but the polarity is reversed.

Just after 270 degrees the impressed voltage once again starts to decrease, and the capacitor must lose electrons from the negative plate. It must discharge, starting at a minimum rate of flow and rising to a maximum. This discharging action continues through the last quarter-cycle (270° to 360°) until the impressed-voltage has reached zero. At 360 degrees you are back at the beginning of the entire cycle, and everything starts over again.

If you examine the complete voltage and current curves in *Figure 1-13, View D*, you will see that the current always arrives at a certain point in the cycle 90 degrees ahead of the voltage because of the charging and discharging action. You know that this time and place relationship between the current and voltage is called the phase relationship. The voltage-current phase relationship in a capacitive circuit is exactly opposite to that in an inductive circuit. The current of a capacitor leads the voltage across the capacitor by 90 degrees.

You realize that the current and voltage are both going through their individual cycles at the same time during the period the ac voltage is impressed. The current does not go through part of its cycle (charging or discharging), stop, and wait for the voltage to catch up. The amplitude and polarity of the voltage and the amplitude and direction of the current are continually changing. Their positions with respect to each other and to the zero line at any electrical instant, any degree between zero and 360 degrees, can be seen by reading upwards from the time-degree line. The current swing from the positive peak at zero degrees to the negative peak at 180 degrees is not a measure of the number of electrons, or the charge on the plates. It is a picture of the direction and strength of the current in relation to the polarity and strength of the voltage appearing across the plates.

At times it is convenient to use the word ICE to recall to mind the phase relationship of the current and voltage in capacitive circuits. I is the symbol for current, and in the word ICE it leads, or comes before, the symbol for voltage, E. C, of course, stands for capacitor. This memory aid is similar to the ELI used to remember the current and voltage relationship in an inductor. The phrase ELI the ICE man is helpful in remembering the phase relationship in both the inductor and capacitor.

Since the plates of the capacitor are changing polarity at the same rate as the ac voltage, the capacitor seems to pass an alternating current. Actually, the electrons do not pass through the dielectric, but their rushing back and forth from plate to plate causes a current flow in the circuit. It is convenient, however, to say that the alternating current flows “through” the capacitor. You know this is not true, but the expression avoids a lot of trouble when speaking of current flow in a circuit containing a capacitor. By the same short cut, you may say that the capacitor does not pass a direct current if both plates are connected to a dc source; current will flow only long enough to charge the capacitor. With a capacitor type of hookup in a circuit containing both ac and dc, only the ac will be “passed” on to another circuit.

You have now learned two things to remember about a capacitor, which are that a capacitor will appear to conduct an alternating current and a capacitor will not conduct a direct current.

4.0.0 REACTANCE and IMPEDANCE

Up to this point inductance and capacitance have been explained individually in ac circuits. This section of the chapter will concern the combination of inductance, capacitance, and resistance in ac circuits.

To explain the various properties that exist within ac circuits, the series RLC circuit will be used. *Figure 1-14* is the schematic diagram of the series RLC circuit. The symbol in *Figure 1-14* that is marked E is the general symbol used to indicate an ac voltage source.

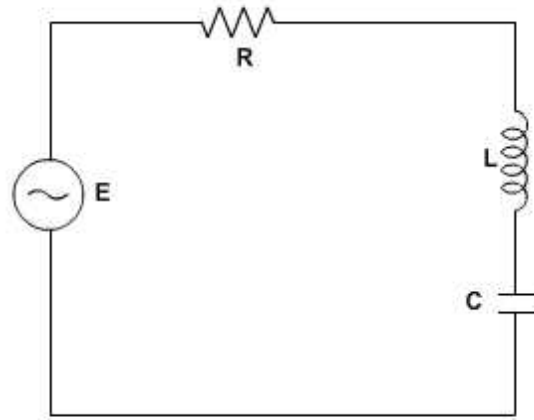


Figure 1-14 – Series RLC circuit.

4.1.0 Reactance

The effect of inductive reactance is to cause the current to lag the voltage, while that of capacitive reactance is to cause the current to lead the voltage. Therefore, since inductive reactance and capacitive reactance are exactly opposite in their effects, what will be the result when the two are combined? It is not hard to see that the net effect is a tendency to cancel each other, with the combined effect then equal to the difference between their values. This resultant is called reactance; it is represented by the symbol X , and is expressed by the equation $X = X_L - X_C$ or $X = X_C - X_L$. Thus, if a circuit contains 50 ohms of inductive reactance and 25 ohms of capacitive reactance in series, the net reactance, or X , is 50 ohms – 25 ohms of inductive reactance.

For a practical example, suppose you have a circuit containing an inductor of 100 μH in a series with a capacitor of .001 μF , and operating at a frequency of 4 MHz. What is the value of net reactance, or X ?

Given: $f = 4\text{MHz}$
 $L = 100 \mu\text{H}$
 $C = .001 \mu\text{F}$

Solution: $X_L = 2\pi fL$
 $X_L = 6.28 \times 4 \text{ MHz} \times 100 \mu\text{H}$
 $X_L = 2512 \Omega$
 $X_C = \frac{1}{2\pi fC}$
 $X_C = \frac{1}{6.28 \times 4 \text{ MHz} \times .001 \mu\text{F}}$
 $X_C = \frac{1}{.02512} \Omega$
 $X_C = 39.8\Omega$
 $X = X_L - X_C$
 $X = 2512 \Omega - 39.8 \Omega$
 $X = 2472.2 \Omega$ (inductive)

Now assume you have a circuit containing a 100 - μH inductor in series with a .0002 - μF capacitor, and operating at a frequency of 1 MHz. What is the value of the resultant reactance in this case?

Given: $f = 1 \text{ MHz}$
 $L = 100 \mu\text{H}$
 $C = .0002 \mu\text{F}$

Solution: $X_L = 2\pi fL$
 $X_L = 6.28 \times 1 \text{ MHz} \times 100 \mu\text{H}$
 $X_L = 628 \Omega$
 $X_C = \frac{1}{2\pi fC}$
 $X_C = \frac{1}{6.28 \times 1\text{MHz} \times .0002 \mu\text{F}}$
 $X_C = \frac{1}{.001256} \Omega$
 $X_C = 796 \Omega$
 $X = X_C - X_L$
 $X = 796 \Omega - 628 \Omega$
 $X = 168 \Omega \text{ (capacitive)}$

You will notice that in this case the inductive reactance is smaller than the capacitive reactance and is therefore subtracted from the capacitive reactance.

These two examples serve to illustrate an important point that when capacitive and inductive reactance are combined in series, the smaller is always subtracted from the larger and the resultant reactance always takes the characteristics of the larger.

4.2.0 Impedance

From your study of inductance and capacitance you know how inductive reactance and capacitive reactance act to oppose the flow of current in an ac circuit. However, there is another factor, the resistance, which also opposes the flow of the current. Since in practice ac circuits containing reactance also contain resistance, the two combine to oppose the flow of current. This combined opposition by the resistance and the reactance is called the impedance, and is represented by the symbol Z.

Since the values of resistance and reactance are both given in ohms, it might at first seem possible to determine the value of the impedance by simply adding them together. It cannot be done so easily. You know that in an ac circuit which contains only resistance, the current and the voltage will be in step (that is, in phase), and will reach their maximum values at the same instant. You also know that in an ac circuit containing only reactance the current will either lead or lag the voltage by one-quarter of a cycle or 90 degrees. Therefore, the voltage in a purely reactive circuit will differ in phase by 90 degrees from that in a purely resistive circuit, and for this reason reactance and resistance are not combined by simply adding them.

When reactance and resistance are combined, the value of the impedance will be greater than either. It is also true that the current will not be in step with the voltage nor will it differ in phase by exactly 90 degrees from the voltage, but it will be somewhere between the in-step and the 90 degree out-of-step conditions. The larger the reactance compared with the resistance, the more nearly the phase difference will approach 90°. The larger the resistance compared to the reactance, the more nearly the phase difference will approach zero degrees.

If the value of resistance and reactance cannot simply be added together to find the impedance, or Z, how is it determined? Because the current through a resistor is in step with the voltage across it and the current in a reactance differs by 90 degrees from the voltage across it, the two are at right angles to each other. They can therefore be combined by means of the same method used in the construction of a right-angle triangle.

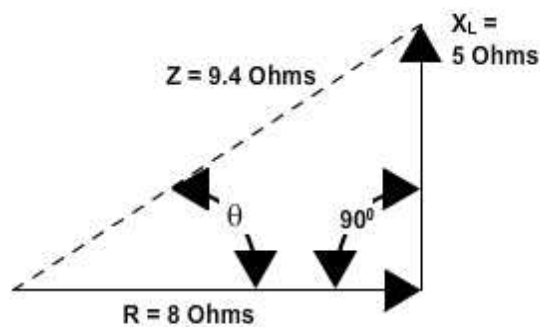


Figure 1-15 – Vector diagram.

Assume you want to find the impedance of a series combination of 8 ohms resistance and 5 ohms inductive reactance. Start by drawing a horizontal line, R, representing 8 ohms resistance, as the base of the triangle. Then, since the effect of the reactance is always at right angles, or 90 degrees, to that of the resistance, draw the line X_L , representing 5 ohms inductive reactance, as the altitude of the triangle. The vector diagram shown in *Figure 1-15* represents the relationship of resistance, inductive reactance, and impedance in a series circuit. Now, complete the hypotenuse (longest side) of the triangle. Then the hypotenuse represents the impedance of the circuit.

One of the properties of a right triangle is:

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{altitude})^2$$

or,

$$\text{hypotenuse} = \sqrt{(\text{base})^2 + (\text{altitude})^2}$$

Applied to impedance, this becomes,

$$(\text{impedance})^2 = (\text{resistance})^2 + (\text{reactance})^2$$

or,

$$\text{impedance} = \sqrt{(\text{resistance})^2 + (\text{reactance})^2}$$

or,

$$Z = \sqrt{R^2 + X^2}$$

Now suppose you apply this equation to check your results in the example given above.

Given: $R = 8\Omega$
 $X_L = 5\Omega$

Solution: $Z = \sqrt{R^2 + X_L^2}$
 $Z = \sqrt{(8\Omega)^2 + (5\Omega)^2}$
 $Z = \sqrt{64 + 25 \Omega}$
 $Z = \sqrt{89 \Omega}$
 $Z = 9.4 \Omega$

When you have a capacitive reactance to deal with instead of inductive reactance as in the previous example, it is customary to draw the line representing the capacitive reactance in a downward direction. *Figure 1-16* is a vector diagram showing relationship of resistance, capacitive reactance, and impedance in a series. The line is drawn downward for capacitive reactance to indicate that it acts in a direction opposite to inductive reactance which is drawn upward. In a series circuit containing capacitive reactance the equation for finding the impedance becomes:

$$Z = \sqrt{R^2 + X_C^2}$$

In many series circuits you will find resistance combined with both inductive reactance and capacitive reactance. Since you know that the value of the reactance, X , is equal to the difference between the values of the inductive reactance, X_L , and the capacitive reactance, X_C , the equation for the impedance in a series circuit containing R , X_L , and X_C then becomes:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

or,

$$Z = \sqrt{R^2 + X^2}$$

In *Figure 1-17* you will see a vector diagram showing relationship of resistance, reactance (capacitive and inductive), and impedance in a series circuit. This method, which may be used to determine the impedance in a series, consists of resistance, inductance, and capacitance.

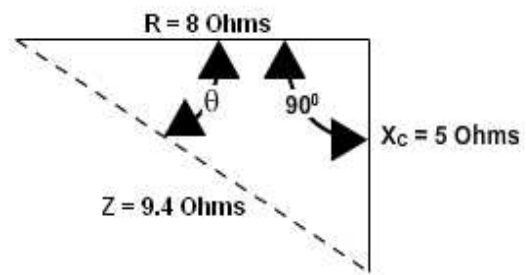
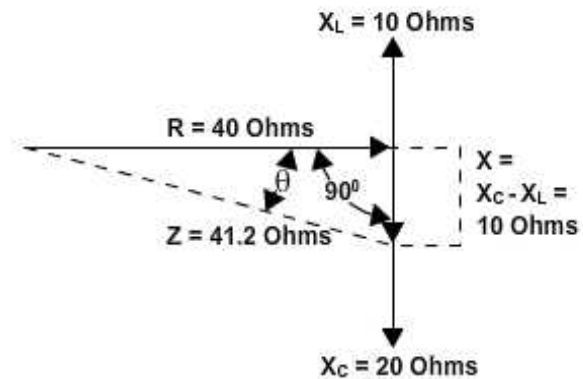


Figure 1-16 – Vector diagram.

Assume that 10 ohms inductive reactance and 20 ohms capacitive reactance are connected in series with 40 ohms resistance. Let the horizontal line represent the resistance R. The line drawn upward from the end of R represents the inductive reactance, X_L . Represent the capacitive reactance by a line drawn downward at right angles from the same end of R. The resultant of X_L and X_C is found by subtracting X_L from X_C . This resultant represents the value of X.



Thus:

$$X = X_C - X_L$$

$$X = 10 \text{ ohms}$$

The line, Z, will then represent the resultant of R and X. The value of Z can be calculated as follows:

Figure 1-17 – Vector diagram.

Given: $X_L = 10 \Omega$

$$X_C = 20 \Omega$$

$$R = 40 \Omega$$

Solution: $X = X_C - X_L$

$$X = 20 \Omega - 10 \Omega$$

$$X = 10 \Omega$$

$$Z = \sqrt{R^2 + X^2}$$

$$Z = \sqrt{(40\Omega)^2 + (10\Omega)^2}$$

$$Z = \sqrt{1600 + 100\Omega}$$

$$Z = \sqrt{1700\Omega}$$

$$Z = 41.2 \Omega$$

4.3.0 OHMS Law for AC

In general, Ohm's law cannot be applied to alternating-current circuits since it does not consider the reactance which is always present in such circuits. However, by a modification of Ohm's law which does take into consideration the effect of reactance we obtain a general law which is applicable to ac circuits. Because the impedance, Z, represents the combined opposition of all the reactances and resistances, this general

law for ac is $I = \frac{E}{Z}$ this general modification applies to alternating current flowing in any circuit, and any one of the values may be found from the equation if the others are known.

For example, suppose a series circuit contains an inductor having 5 ohms resistance and 25 ohms inductive reactance in series with a capacitor having 15 ohms capacitive reactance. If the voltage is 50 volts, what is the current? This circuit can be drawn as shown in *Figure 1-18*.

Given: $R = 5 \Omega$
 $X_L = 25 \Omega$
 $X_C = 15 \Omega$
 $E = 50 \text{ V}$

Solution: $X = X_L - X_C$
 $X = 25 \Omega - 15 \Omega$
 $X = 10 \Omega$

$$Z = \sqrt{R^2 + X^2}$$

$$Z = \sqrt{(5\Omega)^2 + (10\Omega)^2}$$

$$Z = \sqrt{25 + 100 \Omega}$$

$$Z = \sqrt{125 \Omega}$$

$$Z = 11.2 \Omega$$

$$I = \frac{E}{Z}$$

$$I = \frac{50 \text{ V}}{11.2 \Omega}$$

$$I = 4.46 \text{ A}$$

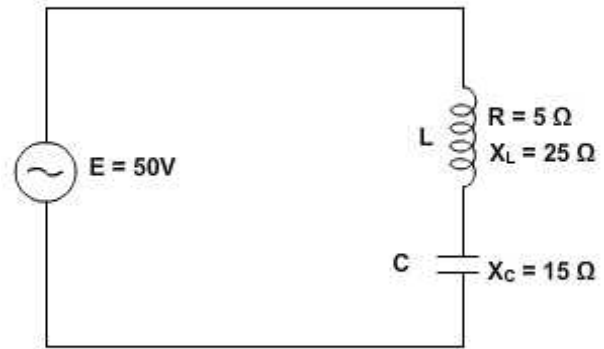


Figure 1-18 – Series LC circuit.

Now suppose the circuit contains an inductor having 5 ohms resistance and 15 ohms inductive reactance in series with a capacitor having 10 ohms capacitive reactance. If the current is 5 amperes, what is the voltage?

Given: $R = 5 \Omega$
 $X_L = 15 \Omega$
 $X_C = 10 \Omega$
 $I = 5 \text{ A}$

Solution: $X = X_L - X_C$
 $X = 15\Omega - 10\Omega$
 $X = 5\Omega$

$$Z = \sqrt{R^2 + X^2}$$

$$Z = \sqrt{(5\Omega)^2 + (5\Omega)^2}$$

$$Z = \sqrt{25 + 25\Omega}$$

$$Z = \sqrt{50\Omega}$$

$$Z = 7.07\Omega$$

$$E = IZ$$

$$E = 5A \times 7.07\Omega$$

$$E = 35.35V$$

5.0.0 LC and RLC CIRCUITS

5.1.0 Series Circuit

The principles and formulas that have been presented in this chapter are use in all ac circuits. The examples given have been series circuits.

This section of the chapter will not present any new material, but will be an example of using all the principles presented so far. You should follow each example problem step by step to see how each formula used depends upon the information determined in earlier steps. When an example calls for solving for square root, you can practice using the square-root table by looking up the values given.

The example series RLC circuit shown in *Figure 1-19* will be used to solve for X_L , X_C , X , Z , I_T , true power, reactive power, apparent power, and power factor. The values solved for will be rounded off to the nearest whole number.

First solve for X_L and X_C .

Given: $f = 60 \text{ Hz}$

$L = 27 \text{ mH}$

$C = 380 \mu\text{F}$

Solution: $X_L = 2 \pi f l$

$$X_L = 6.28 \times 60 \text{ Hz} \times 27 \text{ mH}$$

$$X_L = 10 \Omega$$

$$X_C = \frac{1}{2\pi f c}$$

$$X_C = \frac{1}{6.28 \times 60 \text{ Hz} \times 380 \mu\text{F}}$$

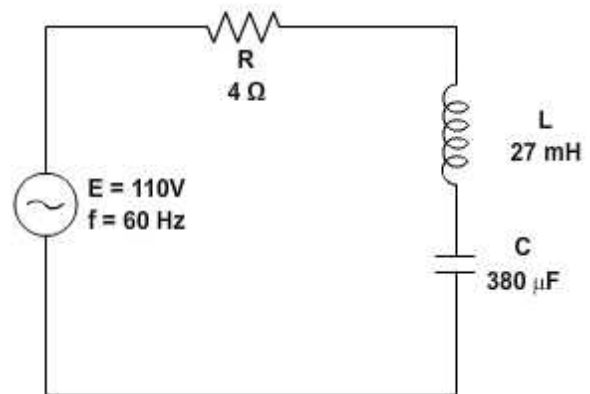


Figure 1-19 – Example series RLC circuit.

$$X_c = \frac{1}{0.143} \Omega$$

$$X_c = 7 \Omega$$

Now solve for X

Given: $X_c = 7 \Omega$

$$X_L = 10 \Omega$$

Solution: $X = X_L - X_c$

$$X = 10 \Omega - 7 \Omega$$

$$X = 3 \Omega \text{ (Inductive)}$$

Use the value of X to solve for Z.

Given: $X = 3 \Omega$

$$R = 4 \Omega$$

Solution: $Z = \sqrt{X^2 + R^2}$

$$Z = \sqrt{(3 \Omega)^2 + (4 \Omega)^2}$$

$$Z = \sqrt{9 + 16} \Omega$$

$$Z = \sqrt{25} \Omega$$

$$Z = 5 \Omega$$

This value of Z can be used to solve for total current (I_T).

Given: $Z = 5 \Omega$

$$E = 110 \text{ V}$$

Solution: $I_T = \frac{E}{Z}$

$$I_T = \frac{110 \text{ V}}{5 \Omega}$$

$$I_T = 22 \text{ A}$$

Since current is equal in all parts of a series circuit, the value of I_T can be used to solve for the various values of power.

Given: $I_T = 22 \text{ A}$

$$R = 4 \Omega$$

$$X = 3 \Omega$$

$$Z = 5 \Omega$$

Solution: True power = $(I_R)^2 R$

$$\text{True power} = (22A)^2 \times 4\Omega$$

$$\text{True power} = 1936 W$$

$$\text{Reactive power} = (I_x)^2 X$$

$$\text{Reactive power} = (22A)^2 \times 3\Omega$$

$$\text{Reactive power} = 1452 \text{ var}$$

$$\text{Apparent power} = (I_z)^2 Z$$

$$\text{Apparent Power} = (22A)^2 \times 5\Omega$$

$$\text{Apparent Power} = 2420 VA$$

The power factor can now be found using either apparent power and true power or resistance and impedance. The mathematics in this example is easier if you use impedance and resistance.

$$\text{Given: } R = 4\Omega$$

$$Z = 5\Omega$$

$$\text{Solution: } PF = \frac{R}{Z}$$

$$PF = \frac{4\Omega}{5\Omega}$$

$$PF = .8 \text{ or } 80 \%$$

5.2.0 Parallel Circuit

When dealing with a parallel ac circuit, you will find that the concepts presented in this chapter for series ac circuits still apply. There is one major difference between a series circuit and a parallel circuit that must be considered. The difference is that current is the same in all parts of a series circuit, whereas voltage is the same across all branches of a parallel circuit. Because of this difference, the total impedance of a parallel circuit must be computed on the basis of the current in the circuit.

You should remember that in the series RLC circuit the following three formulas were used to find reactance, impedance, and power factor:

$$X = X_L - X_C \text{ or } X = X_C - X_L$$

$$Z = \sqrt{(I_R)^2 + X^2}$$

$$PF = \frac{R}{Z}$$

When working with a parallel circuit you must use the following formulas instead:

$$I_x = I_L - I_C \text{ or } I_x = I_C - I_L$$

$$I_z = \sqrt{(I_R)^2 + (I_x)^2}$$

$$PF = \frac{I_R}{I_Z}$$

The impedance of a parallel circuit is found

by the formula $Z = \frac{E}{I_Z}$

NOTE

If no value for E is given in a circuit, any value of E can be assumed to find the values of I_L , I_C , I_X , I_R , and I_Z . The same value of voltage is then used to find impedance.

For example, find the value of Z in the circuit shown in *Figure 1-20*.

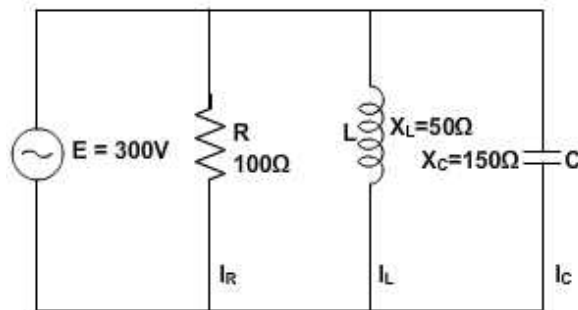


Figure 1-20 – Parallel RLC circuit.

- Given:
- $E = 300\text{ V}$
 - $R = 100\ \Omega$
 - $X_L = 50\ \Omega$
 - $X_C = 150\ \Omega$

The first step in solving for Z is to calculate the individual branch currents.

- Solution:
- $$I_R = \frac{E}{R}$$
- $$I_R = \frac{300\text{ V}}{100\ \Omega}$$
- $$I_R = 3\text{ A}$$
- $$I_L = \frac{E}{X_L}$$
- $$I_L = \frac{300\text{ V}}{50\ \Omega}$$
- $$I_L = 6\text{ A}$$
- $$I_C = \frac{E}{X_C}$$
- $$I_C = \frac{300\text{ V}}{150\ \Omega}$$
- $$I_C = 2\text{ A}$$

Using the values for I_R , I_L , and I_C , solve for I_X and I_Z .

$$I_X = I_L - I_C$$

$$I_X = 6 A - 2 A$$

$$I_X = 4 A \text{ (inductive)}$$

$$I_Z = \sqrt{(I_R)^2 + (I_X)^2}$$

$$I_Z = \sqrt{(3 A)^2 + (4 A)^2}$$

$$I_Z = \sqrt{25 A}$$

$$I_Z = 5 A$$

Using this value of I_Z , solve for Z .

$$Z = \frac{E}{I_Z}$$

$$Z = \frac{300 V}{5 A}$$

$$Z = 60 \Omega$$

If the value for E were not given and you were asked to solve for Z , any value of E could be assumed. If, in the example problem above, you assume a value of 50 volts for E , the solution would be:

Given: $R = 100 \Omega$

$$X_L = 50 \Omega$$

$$X_C = 150 \Omega$$

$$E = 50 V \text{ (assumed)}$$

First solve for the values of current in the same manner as before.

Solution: $I_R = \frac{E}{R}$

$$I_R = \frac{50 V}{100 \Omega}$$

$$I_R = .5 A$$

$$I_L = \frac{E}{X_L}$$

$$I_L = \frac{50 V}{50 \Omega}$$

$$I_L = 1 A$$

$$I_C = \frac{E}{X_C}$$

$$I_C = \frac{50 V}{150 \Omega}$$

$$I_C = .33 A$$

Solve for I_X and I_Z .

$$I_X = I_L - I_C$$

$$I_X = 1A - .33A$$

$$I_X = .67 A (\text{Inductive})$$

$$I_Z = \sqrt{(I_R)^2 + (I_X)^2}$$

$$I_Z = \sqrt{(0.5A)^2 + (0.67A)^2}$$

$$I_Z = \sqrt{0.6989 A}$$

$$I_Z = 0.836 A$$

Solve for Z .

$$Z = \frac{E}{I_Z}$$

$$Z = \frac{50 V}{.836 A}$$

$$Z = 60 \Omega (\text{rounded off})$$

When the voltage is given, you can use the values of currents I_R , I_X , and I_Z , to calculate for the true power, reactive power, apparent power, and power factor. For the circuit shown in *Figure 1-20*, the calculations would be as follows.

To find true power,

Given: $R = 100 \Omega$

$$I_R = 3 A$$

Solution: True power = $(I_R)^2 X$

$$\text{True power} = (3A)^2 \times 100 \Omega$$

$$\text{True power} = 900 W$$

To find reactive power, first find the value of reactance (X).

Given: $E = 300 V$

$$I_X = 4A (\text{Inductive})$$

Solution: $X = \frac{E}{I_x}$

$$X = \frac{300 V}{4 A}$$

$$X = 75 \Omega \text{ (Inductive)}$$

$$\text{Reactive power} = (I_x)^2 X$$

$$\text{Reactive power} = (4 A)^2 \times 75 \Omega$$

$$\text{Reactive power} = 1200 \text{ var}$$

To find apparent power,

Given: $Z = 60 \Omega$

$$I_z = 5 A$$

Solution: $\text{Apparent power} = (I_z)^2 Z$

$$\text{Apparent power} = (5 A)^2 \times 60 \Omega$$

$$\text{Apparent power} = 1500 \text{ VA}$$

The power factor in a parallel circuit is found by either of the following methods.

Given: $\text{True power} = 900 \text{ W}$

$$\text{Apparent power} = 1500 \text{ VA}$$

Solution: $PF = \frac{\text{true power}}{\text{apparent power}}$

$$PF = \frac{900 \text{ W}}{1500 \text{ VA}}$$

$$PF = .6$$

or

Given: $I_R = 3 \text{ A}$

$$I_z = 5 \text{ A}$$

Solution: $PF = \frac{I_R}{I_z}$

$$PF = \frac{3 \text{ A}}{5 \text{ A}}$$

$$PF = .6$$

5.3.0 Series - Parallel Combination

Now that you have seen how series and parallel ac circuit analysis is not fundamentally different than dc circuit analysis, it should come as not surprise that series-parallel analysis would be the same as well, just using complex numbers instead of *scalar* to represent voltage, current, and impedance. *Figure 1-21* is an example of a series-parallel circuit.

The first order of business, as usual, is to determine values of impedance (Z) for all components based on the frequency of the ac power source. To do this, you need to first determine values of reactance (X) for all inductors and capacitors, then convert reactance (X) and resistance (R) figures into proper impedance (Z) form (*Table 1-1*):

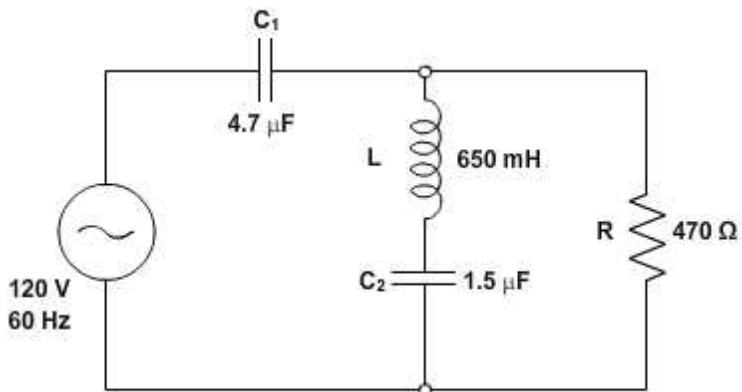


Figure 1-21 — Example series-parallel R, L, and C circuit.

Table 1-1—Reactances and Resistances.

$X_{C1} = \frac{1}{2\pi f C_1}$	$X_L = 2\pi f L$
$X_{C1} = \frac{1}{(2)(\pi)(60 \text{ Hz})(4.7 \mu\text{F})}$	$X_L = (2)(\pi)(60 \text{ Hz})(650 \text{ mH})$
$X_{C1} = 565 \Omega$	$X_L = 245 \Omega$
$X_{C2} = \frac{1}{2\pi f C_2}$	
$X_{C2} = \frac{1}{(2)(\pi)(60 \text{ Hz})(1.5 \mu\text{F})}$	$R = 470 \Omega$
$X_{C2} = 1768 \Omega$	

$$Z_{C1} = 0 - j564.38 \Omega \text{ or } 564.38 \Omega \angle -90^\circ$$

$$Z_L = 0 + j245.04 \Omega \text{ or } 245.04 \Omega \angle 90^\circ$$

$$Z_{C2} = 0 - j1.7684k \Omega \text{ or } 1.7684 k\Omega \angle -90^\circ$$

$$Z_R = 470 + j0 \Omega \text{ or } 470 \Omega \angle 0^\circ$$

Now you can set up the initial values in *Table 1-2*:

Table 1-2–Initial Values.

	C_1	L	C_2	R	<i>Total</i>	
<i>E</i>					120 + j0 120 \angle 0°	<i>Volts</i>
<i>I</i>						<i>Amps</i>
<i>Z</i>	0 – j564.38 564.38 \angle –90°	0 + j245.04 245.04 \angle 90°	0 – j1.7684L 1.7684L \angle –90°	470 + j0 470 \angle 0°		<i>Ohms</i>

Because it is a series-parallel combination circuit, you must reduce it to a total impedance in more than one step. The first step is to combine L and C_2 as a series combination of impedances by adding their impedances together. Then, that impedance will be combined in parallel with the impedance of the resistor to arrive at another combination of impedances. Finally, that quantity will be added to the impedance of C_1 to arrive at the total impedance.

Table 1-3–Reciprocal Formula.

	$L--C_2$	$R//(L--C_2)$	<i>Total</i> $C_1--[R//(L--C_2)]$	
<i>E</i>			120 + j0 120 \angle 0°	<i>Volts</i>
<i>I</i>				<i>Amps</i>
<i>Z</i>	0 – j1.5233k 1.5233k \angle –90°	429.15 – j132.41 449.11 \angle –17.147°	429.15 – j696.79 818.34 \angle –58.371°	<i>Ohms</i>

↑

Rule of series circuits:

$$Z_{L--C2} = Z_L + Z_{C2}$$

↑

Rule of parallel circuits:

$$Z_{R//(L--C2)} = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_{L--C2}}}$$

↑

Rule of series circuits:

$$Z_{TOTAL} = Z_{C1} + Z_{R/(L--C2)}$$

-- represents series

// represents parallel

In order that your table may follow all these steps, it will be necessary to add additional columns to it so that each step may be represented. Adding more columns horizontally to *Table 1-2* would be impractical for formatting reasons, so a new row of columns has been placed underneath each column designated by its respective component combination in *Table 1-3*. Calculating these new (combination) impedances will require complex addition for series combinations, and the “reciprocal” formula for complex impedances in parallel. This time, there is no avoidance of the reciprocal formula; the required figures can be arrived at no other way.

Because *Table 1-3* contains a column for “Total,” we can safely discard that column from *Table 1-2*. This gives you one table with four columns and another table with three columns. Now that you know the total impedance ($818.34 \Omega \angle -58.371^\circ$) and the total voltage ($120 \text{ volts } \angle 0^\circ$), you can apply Ohm’s Law ($I = E / Z$) vertically in the “Total” column to arrive at a figure for total current:

Table 1-3–Reciprocal Formula


	$L - -C_2$	$R // (L - -C_2)$	<i>Total</i> $C_1 - - [R // (L - -C_2)]$	
<i>E</i>			120 + j0 120 $\angle 0^\circ$	<i>Volts</i>
<i>I</i>			76.899m + j124.86m 146.64m $\angle 58.371^\circ$	<i>Amps</i>
<i>Z</i>	0 – j1.5233k 1.5233k $\angle -90^\circ$	429.15 – j132.41 449.11 $\angle -17.147^\circ$	429.15 – j696.79 818.34 $\angle -58.371^\circ$	<i>Ohms</i>

↑
Ohm’s
Law
 $I = \frac{E}{Z}$

At this point you ask yourself the question: are there any components or component combinations which share either the total voltage or the total current? In this case, both C_1 and the parallel combination $R // (L - -C_2)$ share the same (total) current, since the total impedance is composed of the two sets of impedances in series. Thus, you can transfer the figure for total current into both columns (*Tables 1-4* and *1-5*):

Table 1-4–Rule of Series Circuits.

	C_1	L	C_2	R	
E					Volts
I	$76.899m + j124.86m$ $146.64m \angle 58.371^\circ$				Amps
Z	$0 - j564.38$ $564.38 \angle -90^\circ$	$0 + j245.04$ $245.04 \angle 90^\circ$	$0 - j1.7684k$ $1.7684k \angle -90^\circ$	$470 + j0$ $470 \angle 0^\circ$	Ohms




Rule of series circuits:


$$I_{total} = I_{C1} = I_{R//(L-C2)}$$

Table 1-5–Reciprocal Formula Updated.

	$L--C_2$	$R//(L--C_2)$	Total $C_1--[R//(L--C_2)]$	
E			$120 + j0$ $120 \angle 0^\circ$	Volts
I		$76.899m + j124.86m$ $146.64m \angle 58.371^\circ$	$76.899m + j124.86m$ $146.64m \angle 58.371^\circ$	Amps
Z	$0 - j1.5233k$ $1.5233k \angle -90^\circ$	$429.15 - j132.41$ $449.11 \angle -17.147^\circ$	$429.15 - j696.79$ $818.34 \angle -58.371^\circ$	Ohms



Rule of series circuits:

$$I_{Total} = I_{C1} = I_{R//(L-C2)}$$


Now, you can calculate voltage drops across C_1 and the series-parallel combination of $R//(L--C_2)$ using Ohm's Law ($E = IZ$) vertically in those table columns (Figures 1-6 and 1-7).

Table 1-6–Rule of Series Circuits Updated.

	C_1	L	C_2	R	
E	70.467 – j43.400 82.760 \angle – 31.629°				Volts
I	76.899m + j124.86m 146.64m \angle 58.371°				Amps
Z	0 – j564.38 564.38 \angle – 90°	0 + j245.04 245.04 \angle 90°	0 – j1.7684k 1.7684k \angle – 90°	470 + j0 470 \angle 0°	Ohms

↑
Ohm's Law
 $E = IZ$

1-7–Reciprocal Formula Updated.

	$L -- C_2$	$R // (L -- C_2)$	Total $C_1 -- [R // (L -- C_2)]$	
E		49.533 + j43.400 65.857 \angle 41.225°	120 + j0 120 \angle 0°	Volts
I		76.899m + j124.86m 146.64m \angle 58.371°	76.899m + j124.86m 146.64m \angle 58.371°	Amps
Z	0 – j1.5233k 1.5233k \angle – 90°	429.15 – j132.41 449.11 \angle – 17.147°	429.15 – j696.79 818.34 \angle – 58.371°	Ohms

↑
Ohm's Law
 $E = IZ$

A quick double-check of your work at this point would be to see whether or not the voltage drops across C_1 and the series-parallel combination of $R // (L -- C_2)$ indeed add up to the total. According to Kirchhoff's Voltage Law, they should be correct.

$$E_{total} \text{ should be equal to } E_{C_1} + E_{R // (L -- C_2)}$$

$$\begin{array}{r} 70.467 - j43.400V \\ + 49.533 + j43.400V \\ \hline 120 + j0V \end{array}$$

That last step was merely a precaution. In a problem with as many steps as this one has, there is much opportunity for error. Occasional cross-checks like the one above can save you a lot of work and unnecessary frustration by identifying problems prior to the final step of the problem.

After having solved for voltage drops across C_1 and the combination $R//(L--C_2)$, you can again ask yourself the question: what other components share the same voltage or current? In this case, the resistor (R) and the combination of the inductor and the second capacitor ($L--C_2$) share the same voltage because those sets of impedances are in parallel with each other. Therefore, you can transfer the voltage figure just solved for into the columns for R and $L--C_2$ (Tables 1-8 and 1-9).

Table 1-8--Rule of Series Circuits Updated.

	C_1	L	C_2	R	
E	70.467 - j43.400 82.760 \angle -31.629°			49.533 + j43.400 65.857 \angle 41.225°	Volts
I	76.899m + j124.86m 146.64m \angle 58.371°				Amps
Z	0 - j564.38 564.38 \angle -90°	0 + j245.04 245.04 \angle 90°	0 - j1.7684k 1.7684k \angle -90°	470 + j0 470 \angle 0°	Ohms

Rule of parallel circuits:

$$E_{R//(L-C_2)} = E_R = E_{L-C_2}$$

Table 1-9--Reciprocal Formula Updated.

	$L--C_2$	$R//(L--C_2)$	Total $C_1--[R//(L--C_2)]$	
E	49.533 + j43.400 65.857 \angle 41.225°	49.533 + j43.400 65.857 \angle 41.225°	120 + j0 120 \angle 0°	Volts
I		76.899m + j124.86m 146.64m \angle 58.371°	76.899m + j124.86m 146.64m \angle 58.371°	Amps
Z	0 - j1.5233k 1.5233k \angle -90°	429.15 - j132.41 449.11 \angle -17.147°	429.15 - j696.79 818.34 \angle -58.371°	Ohms

Rule of parallel circuits:

$$E_{R//(L-C_2)} = E_R = E_{L-C_2}$$

Now you are all set for calculating current through the resistor and through the series combination L--C₂. All you need to do is apply Ohm's Law ($I=E/Z$) vertically in both of those columns (Tables 1-10 and 1-11).

Table 1-10--Rule of Series Circuits Updated.

	C ₁	L	C ₂	R	
E	70.467 – j43.400 82.760 ∠ – 31.629°			49.533 + j43.400 65.857 ∠ 41.225°	Volts
I	76.899m + j124.86m 146.64m ∠ 58.371°			105.39m + j92.341m 140.12m ∠ 41.225°	Amps
Z	0 – j564.38 564.38 ∠ – 90°	0 + j245.04 245.04 ∠ 90°	0 – j1.7684k 1.7684k ∠ – 90°	470 + j0 470 ∠ 0°	Ohms

↑
Ohm's Law
 $I = EZ$

Table 1-11--Reciprocal Formula Updated.

	$L--C_2$	$R//(L--C_2)$	Total $C_1--[R//(L--C_2)]$	
E	49.533 + j43.400 65.857 ∠ 41.225°	49.533 + j43.400 65.857 ∠ 41.225°	120 + j0 120 ∠ 0°	Volts
I	– 28.490m + j32.516m 43.232m ∠ 131.22°	76.899m + j124.86m 146.64m ∠ 58.371°	76.899m + j124.86m 146.64m ∠ 58.371°	Amps
Z	0 – j1.5233k 1.5233k ∠ – 90°	429.15 – j132.41 449.11 ∠ – 17.147°	429.15 – j696.79 818.34 ∠ – 58.371°	Ohms

↑
Ohm's Law
 $I = EZ$

Another quick double-check of your work at this point would be to see if the current figures for L -- C₂ and R add up to the total current. According to Kirchoff's Current Law, they should be correct.

$$\begin{aligned}
 &I_{R//(L--C_2)} \text{ should be equal to } I_R + I_{(L--C_2)} \\
 &105.39m + j92.341m \\
 &+ - 28.490m + j32.516m \\
 \hline
 &76.899m + j124.86m
 \end{aligned}$$

Since the L and C₂ are connected in series, and since you know the current through their series combination impedance, you can distribute that current figure to the L and C₂ columns following the rule of series circuits whereby series components share the same current (Table 1-12).

Table 1-12–Rule of Series Circuits Updated.

	C ₁	L	C ₂	R	
E	70.467 – j43.400 82.760 ∠ – 31.629°			49.533 + j43.400 65.857 ∠ 41.225°	Volts
I	76.899m + j124.86m 146.64m ∠ 58.371°	– 28.490m + 32.516m 43.232m ∠ 131.22°	– 28.490m + j32.516m 43.232m ∠ 131.22°	105.39m + j92.341m 140.12m ∠ 41.225°	Amps
Z	0 – j564.38 564.38 ∠ – 90°	0 + j245.04 245.04 ∠ 90°	0 – j1.7684k 1.7684k ∠ – 90°	470 + j0 470 ∠ 0°	Ohms

Rule of series circuits:

$$I_{L-C_2} = I_L = I_{C_2}$$

With one last step (actually, two calculations), you can complete your analysis table for this circuit. With impedance and current figures in place for L and C₂, all you have to do is apply Ohm's Law (E=IZ) vertically in those two columns to calculate voltage drops (Table 1-13).

Table 1-13–Rule of Series Circuits Updated.

	C ₁	L	C ₂	R	
E	70.467 – j43.400 82.760 ∠ – 31.629°	– 7.968 – j6.981 10.594 ∠ 221.22°	57.501 + j50.382 76.451 ∠ 41.225	49.533 + j43.400 65.857 ∠ 41.225°	Volts
I	76.899m + j124.86m 146.64m ∠ 58.371°	– 28.490m + 32.516m 43.232m ∠ 131.22°	– 28.490m + j32.516m 43.232m ∠ 131.22°	105.39m + j92.341m 140.12m ∠ 41.225°	Amps
Z	0 – j564.38 564.38 ∠ – 90°	0 + j245.04 245.04 ∠ 90°	0 – j1.7684k 1.7684k ∠ – 90°	470 + j0 470 ∠ 0°	Ohms

↑
Ohm's Law
 $E = IZ$

↑
Ohm's Law
 $E = IZ$

6.0.0 POWER IN AC CIRCUITS

6.1.0 Impedance

In a purely resistive circuit all of the power is consumed and none is returned to the source; in a purely reactive circuit no power is consumed and all of the power is returned to the source. It follows that in a circuit which contains both resistance and reactance there must be some power dissipated in the resistance as well as some returned to the source by the reactance. In *Figure 1-22* you can see the relationship between the voltage, the current, and the power in such a circuit. The part of the power curve which is shown below the horizontal reference line is the result of multiplying a positive instantaneous value of current by a negative instantaneous value of the voltage, or vice versa. As you know, the product obtained by multiplying a positive value by a negative value will be negative. Therefore the power at that instant must be considered as negative power. In other words, during this time the reactance was returning power to the source.

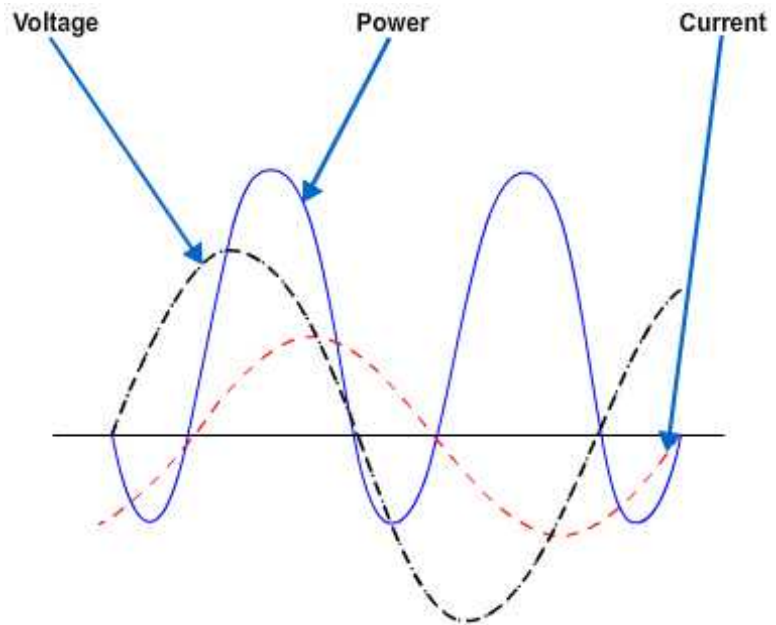


Figure 1-22 — Instantaneous power when current and voltage are out of phase.

The instantaneous power in the circuit is equal to the product of the applied voltage and current through the circuit. When the voltage and current are of the same polarity they are acting together and taking power from the source. When the polarities are unlike, they are acting in opposition and power is being returned to the source. Briefly then, in an ac circuit which contains reactance as well as resistance, the apparent power is reduced by the power returned to the source, so that in such a circuit the net power, or true power, is always less than the apparent power.

6.2.0 Total Current

You know that in a direct current circuit the power is equal to the voltage times the current, or $P = E \times I$. If a voltage of 100 volts applied to a circuit produces a current of 10 amperes, the power is 1000 watts. This is also true in an ac circuit when the current and voltage are in phase, that is, when the circuit is effectively resistive. But, if the ac circuit contains reactance, the current will lead or lag the voltage by a certain amount (the phase angle). When the current is out of phase with the voltage, the power indicated by the product of the applied voltage and the total current gives only what is known as the apparent power. The true power depends upon the phase angle between the current and voltage. The symbol for phase angle is θ (theta).

6.3.0 Voltage

When an alternating voltage is impressed across a capacitor, power is taken from the source and stored in the capacitor as the voltage increases from zero to its maximum value. Then, as the impressed voltage decreases from its maximum value to zero, the capacitor discharges and returns the power to the source. Likewise, as the current through an inductor increases from its zero value to its maximum value, the field around the inductor builds up to a maximum, and when the current decreases from maximum to zero, the field collapses and returns the power to the source. You can see, therefore, that no power is used up in either case, since the power alternately flows to and from the source. This power that is returned to the source by the reactive components in the circuit is called reactive power.

6.4.0 True Power

As mentioned before, the true power of a circuit is the power actually used in the circuit. This power, measured in watts, is the power associated with the total resistance in the circuit. To calculate true power, the voltage and current associated with the resistance must be used. Since the voltage drop across the resistance is equal to the resistance multiplied by the current through the resistance, true power can be calculated by the formula: True power = $(I_R)^2 R$

Where: True power is measured in watts I_R is resistive current in amperes R is resistance in ohms

For example, find the true power of the circuit in *Figure 1-23*.

Given: $R = 60\Omega$
 $X_L = 30\Omega$
 $X_C = 110\Omega$
 $E = 500V$

Solution: $X = X_C - X_L$
 $X = 110\Omega - 30\Omega$
 $X = 80\Omega$

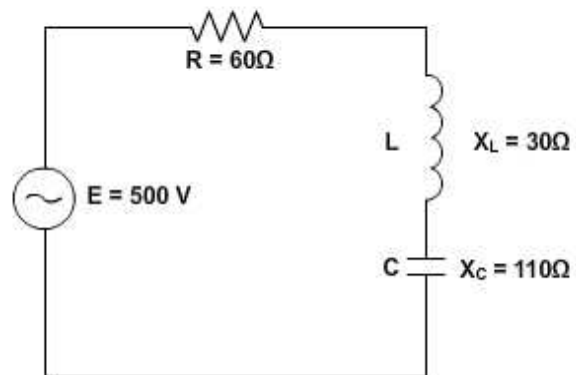


Figure 1-23 – Example circuit for determining power.

$$Z = \sqrt{R^2 + X^2}$$

$$Z = \sqrt{(60\Omega)^2 + (80\Omega)^2}$$

$$Z = \sqrt{3600 + 6400\Omega}$$

$$Z = \sqrt{10,000\Omega}$$

$$Z = 100 \Omega$$

$$I = \frac{E}{Z}$$

$$I = \frac{500 V}{100 \Omega}$$

$$I = 5 A$$

Since the current in a series circuit is the same in all parts of the circuit:

$$\text{True power} = (I_R)^2 R$$

$$\text{True power} = (5A)^2 \times 60 \Omega$$

$$\text{True power} = 1500 \text{ watts}$$

6.5.0 Reactive Power

The reactive power is the power returned to the source by the reactive components of the circuit. This type of power is measured in Volt-Amperes-Reactive, abbreviated (var). Reactive power is calculated by using the voltage and current associated with the circuit reactance. Since the voltage of the reactance is equal to the reactance multiplied by the reactive current, reactive power can be calculated by the formula:

$$\text{Reactive power} = (I_X)^2 X$$

Where: Reactive power is measured in volt-amperes-reactive.

I_X is reactive current in amperes.

X is total reactance in ohms.

Another way to calculate reactive power is to calculate the inductive power and capacitive power and subtract the smaller from the larger.

$$\text{Reactive power} = (I_L)^2 X_L - (I_C)^2 X_C$$

or,

$$(I_C)^2 X_C - (I_L)^2 X_L$$

Where: Reactive power is measured in volt-amperes-reactive.

I_C is capacitive current in amperes.

X_C is capacitive reactance in ohms.

6.6.0 Apparent Power

Apparent power is the power that appears to the source because of the circuit impedance. Since the impedance is the total opposition to ac, the apparent power is that power the voltage source “sees.” Apparent power is the combination of true power and reactive power. Apparent power is not found by simply adding true power and reactive power just as impedance is not found by adding resistance and reactance. To calculate apparent power, you may use either of the following formulas:

$$\text{Apparent power} = (I_z)^2 Z$$

Where: Apparent power is measured in VA (*volt – amperes*).

I_z is impedance current in amperes.

Z is impedance in ohms.

or,

$$\text{Apparent power} = \sqrt{(\text{True power})^2 + (\text{reactive power})^2}$$

For example, find the apparent power for the circuit shown in *Figure 1-23*.

Given: $Z = 100 \Omega$

$I = 5A$

Recall that current in a series circuit is the same in all parts of the circuit.

Solution:

$$\text{Apparent power} = (I_z)^2 Z$$

$$\text{Apparent power} = (5A)^2 \times 100 \Omega$$

$$\text{Apparent power} = 2,500 \text{ VA}$$

or,

Given: True power = 1,500 W

Reactive power = 2,000 var

$$\text{Apparent power} = \sqrt{(\text{True power})^2 + (\text{reactive power})^2}$$

$$\text{Apparent power} = \sqrt{(1,500 \text{ W})^2 + (2,000 \text{ var})^2}$$

$$\text{Apparent power} = \sqrt{625 \times 10^4 \text{ VA}}$$

$$\text{Apparent power} = 2,500 \text{ VA}$$

I_L is inductive current in amperes.

X_L is inductive reactance in ohms.

Either one of these formulas will work. The formula you use depends upon the values you are given in a circuit. For example, find the reactive power of the circuit shown in *Figure 1-23*.

Given: $X_L = 30 \Omega$

$$X_C = 110 \Omega$$

$$X = 80 \Omega$$

$$I = 5 A$$

Since this is a series circuit, current (I) is the same in all parts of the circuit.

Solution: Reactive power = $(I_X)^2 X$

$$\text{Reactive power} = (5A)^2 \times 80 \Omega$$

$$\text{Reactive power} = 2,000 \text{ var}$$

You can see by the following that the second formula also works.

$$\text{Reactive power} = (I_C)^2 X_C - (I_L)^2 X_L$$

$$\text{Reactive power} = (5A)^2 \times 110 \Omega - (5A)^2 \times 30 \Omega$$

$$\text{Reactive power} = 2,750 \text{ var} - 750 \text{ var}$$

$$\text{Reactive power} = 2,000 \text{ var}$$

7.0.0 POWER FACTOR

The power factor is a number (represented as a decimal or a percentage) that represents the portion of the apparent power dissipated in a circuit. If you are familiar with trigonometry, the easiest way to find the power factor is to find the cosine of the phase angle (θ). The cosine of the phase angle is equal to the power factor. You do not need to use trigonometry to find the power factor. Since the power dissipated in a circuit is true power, then:

$$\text{Apparent Power} \times \text{PF} = \text{True Power. Therefore, } \text{PF} = \frac{\text{True Power}}{\text{Apparent Power}}.$$

If true power and apparent power are known, you can use this formula. Going one step further, another formula for power factor can be developed. By substituting the equations for true power and apparent power in the formula for power factor, you get:

$$\text{PF} = \frac{(I_R)^2 R}{(I_Z)^2 Z}$$

Since current in a series circuit is the same in all parts of the circuit, I_R equals I_Z .

Therefore, in a series circuit, $PF = \frac{R}{Z}$.

For example, to compute the power factor for the series circuit shown in *Figure 1-23*, any of the above methods may be used.

Given: True power = 1,500 W
Apparent power = 2,500 VA

Solution: $PF = \frac{\text{True Power}}{\text{Apparent Power}}$

$$PF = \frac{1,500 \text{ W}}{2,500 \text{ VA}}$$

$$PF = .6$$

Another method:

Given: $R = 60 \Omega$
 $Z = 100 \Omega$

Solution: $PF = \frac{R}{Z}$

$$PF = \frac{60 \Omega}{100 \Omega}$$

$$PF = .6$$

NOTE

As stated earlier the power factor can be expressed as a decimal or percentage. In the examples above the decimal number .6 could be expressed as 60%.

7.1.0 Power Factor Correction

The apparent power in an ac circuit has been described as the power the source “sees.” As far as the source is concerned, the apparent power is the power that must be provided to the current. You also know that the true power is the power actually used in the circuit. The difference between apparent power and true power is wasted because, in reality, only true power is consumed. The ideal situation would be for apparent power and true power to be equal. If this were the case, the power factor would be 1 (unity) or 100 percent. There are two ways in which this condition can exist: (1) if the circuit is purely resistive or (2) if the circuit “appears” purely resistive to the source. To make the circuit appear purely resistive, there must be no reactance. To have no reactance in the circuit, the inductive reactance (X_L) and capacitive reactance (X_C) must be equal.

Remember: $X = X_L - X_C$, therefore when $X_L = X_C$ $X = 0$. The expression “correcting the power factor” refers to reducing the reactance in a circuit. The ideal situation is to have no reactance in the circuit. This is accomplished by adding capacitive reactance to a circuit which is inductive and inductive reactance to a circuit which is capacitive. For example, the circuit shown in *Figure 1-23* has a total reactance of 80 ohms capacitive and the power factor was .6 or 60 percent. If 80 ohms of inductive reactance were

added to this circuit (by adding another inductor), the circuit would have a total reactance of zero ohms and a power factor of 1 or 100 percent. The apparent and true power of this circuit would then be equal.

Summary

Your knowledge, understanding, and application of advanced electrical theory are very important for the safe conduct and completion of your job as a Construction Electrician. As a Construction Electrician, you need the knowledge of the concepts and principles when dealing with alternating and direct current. During your career as a Construction Electrician, you will apply this and other electrical and electronic theory in your everyday conduct.