

## CHAPTER 4

## GPS Absolute or Autonomous Point Positioning Determination Concepts, Errors, and Accuracies

4-1. General. As outlined in Chapter 2, the GPS was originally conceived and designed to provide point positioning and velocity of a user with a single, usually low-cost, hand-held GPS receiver. This is termed "absolute" or "autonomous" point positioning, as distinguished from "relative" positioning when a second receiver is employed. GPS absolute or autonomous positioning is the most widely used military and commercial GPS positioning method for real-time navigation and location. It is usually not sufficiently accurate for precise surveying, mapping, or hydrographic positioning uses (given a total UERE or user equivalent range error of 5.2 m 1-sigma or 10.3 m 95%, expected horizontal position accuracy is 9.4 m 95% and expected vertical position accuracy is 17.3 m 95%. Refer to *Global Positioning System Standard Positioning Service Performance Standard, 4th Edition, September 2008*, which may be accessed at <http://pnt.gov/public/docs/> for more detailed information.) However, there are numerous other Corps applications where absolute/autonomous point positioning is sufficiently accurate: vessel/vehicle/personnel navigation, emergency operations, reconnaissance mapping, dredge disposal monitoring, etc. This chapter discusses the general concepts of performing absolute/autonomous point positioning, and some of the basic errors inherent in the process.

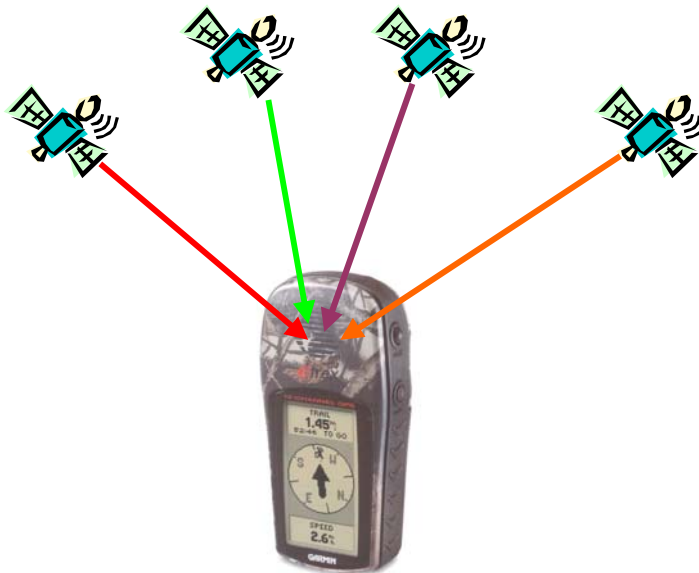


Figure 4-1. Point positioning range measurements from a passive hand-held GPS receiver

4-2. Absolute or Autonomous Point Positioning. Absolute/autonomous positioning involves the use of only a single passive receiver at the user's location to collect data from multiple satellites in order to determine the user's georeferenced position--see Figure 4-1. GPS determination of a point position on the earth actually uses a technique common to terrestrial surveying called trilateration--i.e. electronic distance measurement resection. The user's GPS receiver simply measures the distance (i.e. ranges) between the earth and the GPS satellites. The user's position is determined by the resected intersection of the observed ranges to the satellites. At least 4 satellite ranges are required to compute a 3-D position and resolve timing variations between the satellite and receiver clocks. Adding more satellite ranges will provide redundancy (and more accuracy) in the position solution. The resultant 3-D coordinate value is relative to the geocentric reference system (WGS84). The GPS receiver may be operated in a static or dynamic mode. The accuracies obtained by GPS absolute/autonomous positioning are dependent on the user's GPS receiver quality, location, and length of observation time, Dilution of Precision (DOP), and many other factors. Future GPS satellite modernization upgrades, enhanced code and carrier processing techniques, and other refinements are expected to significantly improve the accuracy of absolute/autonomous positioning such that meter-level navigation accuracies may be available in real-time.

4-3. GPS Absolute or Autonomous Position Solution Process—Pseudoranging. When a GPS user performs a navigation solution, only an approximate range, or "pseudorange," to selected satellites is measured. In order for the GPS user to determine their precise location, the range to the satellite must be measured and the position of those satellites must be known. By pseudoranging, the GPS user measures an approximate distance between the GPS antenna and the satellite by correlation of a satellite-transmitted code and a reference code created by the receiver. This measurement does not contain corrections for synchronization errors between the clock of the satellite transmitter and that of the GPS receiver. The distance the signal has traveled is equal to the velocity of the transmission multiplied by the elapsed time of transmission. The signal velocity is affected by tropospheric and ionospheric conditions in the atmosphere. Figure 4-2 illustrates this pseudoranging concept.

a. The accuracy of the positioned point is a function of the range measurement accuracy and the geometry of the satellites, as reduced to spherical intersections with the earth's surface. A description of the geometrical magnification of uncertainty in a GPS determined point position is termed "Dilution of Precision" (DOP), which is discussed in a later section. Repeated and redundant range observations will generally improve range accuracy. However, the dilution of precision remains the same. In a static mode (meaning the GPS receiver antenna stays stationary), range measurements to each satellite may be continuously remeasured over varying orbital locations of the satellites. The varying satellite orbits cause varying positional intersection geometry. In addition, simultaneous range observations to numerous satellites can be adjusted using weighting techniques based on the elevation and pseudorange measurement reliability.

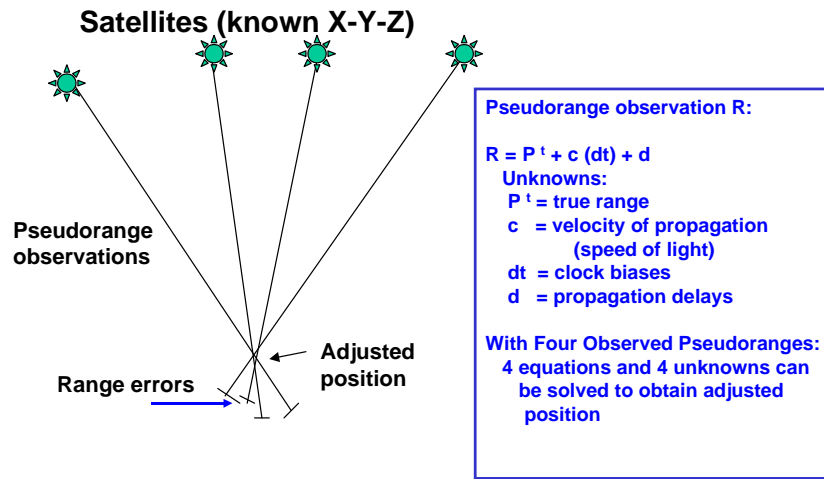


Figure 4-2. Pseudoranging technique

b. Four pseudorange observations are needed to resolve a GPS 3-D position. (Only three pseudorange observations are needed for a 2-D location.) In practice there are often more than four satellites within view. A minimum of four satellite ranges are needed to resolve the clock biases contained in both the satellite and the ground-based receiver. Thus, in solving for the X-Y-Z coordinates of a point, a fourth unknown (i.e. clock bias-- $\Delta t$ ) must also be included in the solution. The solution of the 3-D position of a point is simply the solution of four pseudorange observation equations containing four unknowns, i.e. X, Y, Z, and  $\Delta t$ .

c. A pseudorange observation is equal to the true range from the satellite to the user plus delays due to satellite/receiver clock biases and other effects.

$$R = p^t + c(\Delta t) + d \quad (\text{Eq 4-1})$$

where

- $R$  = observed pseudorange
- $p^t$  = true range to satellite (unknown)
- $c$  = velocity of propagation
- $\Delta t$  = clock biases (receiver and satellite)
- $d$  = propagation delays due to atmospheric conditions

Propagation delays ( $d$ ) are usually estimated from atmospheric models.

The true range " $p^t$ " is equal to the 3-D coordinate difference between the satellite and user.

$$p^t = [(X^s - X^u)^2 + (Y^s - Y^u)^2 + (Z^s - Z^u)^2]^{1/2} \quad (\text{Eq 4-2})$$

where

$X^s, Y^s, Z^s$  = known satellite geocentric coordinates from ephemeris data

$X^u, Y^u, Z^u$  = unknown geocentric coordinates of the user which are to be determined.

When four pseudoranges are observed, four equations are formed from Equations 4-1 and 4-2.

$$(R_1 - c \Delta t - d_1)^2 = (X_1^s - X^u)^2 + (Y_1^s - Y^u)^2 + (Z_1^s - Z^u)^2 \quad (\text{Eq 4-3})$$

$$(R_2 - c \Delta t - d_2)^2 = (X_2^s - X^u)^2 + (Y_2^s - Y^u)^2 + (Z_2^s - Z^u)^2 \quad (\text{Eq 4-4})$$

$$(R_3 - c \Delta t - d_3)^2 = (X_3^s - X^u)^2 + (Y_3^s - Y^u)^2 + (Z_3^s - Z^u)^2 \quad (\text{Eq 4-5})$$

$$(R_4 - c \Delta t - d_4)^2 = (X_4^s - X^u)^2 + (Y_4^s - Y^u)^2 + (Z_4^s - Z^u)^2 \quad (\text{Eq 4-6})$$

In these equations, the only unknowns are  $X^u, Y^u, Z^u$ , and  $\Delta t$ . Solving these four equations for the four unknowns at each GPS update yields the user's 3-D position coordinates-- $X^u, Y^u, Z^u$ . These geocentric coordinates can then be transformed to any user reference datum. Adding more pseudorange observations provides redundancy to the solution. For instance, if seven satellites are simultaneously observed, seven equations are derived and still only four unknowns result.

d. This solution quality is highly dependent on the accuracy of the known coordinates of each satellite (i.e.  $X^s, Y^s$ , and  $Z^s$ ), the accuracy with which the atmospheric delays "d" can be estimated through modeling, and the accuracy of the resolution of the actual time measurement process performed in a GPS receiver (clock synchronization, signal processing, signal noise, etc.). As with any measurement process, repeated and long-term observations from a single point will enhance the overall positional reliability.

4-4. GPS Point Positioning Accuracies. Determining the accuracy of a point position derived from GPS observations is a complex and highly variable process. Any specified accuracy (or claimed accuracy) is subject to many qualifications and interpretations. This is due to the numerous components that make up the "error budget" of a GPS observation. As an example, the Standard Positioning Service or SPS GPS position accuracy statistics for the combination of: a 4.0 m 1-sigma (7.8 m 95%) SIS-only (signal-in-space only) URE (user range error) value over all AODs (age of data) during normal operations, the modern single-frequency SPS receiver UEE (user equipment error) of 2.3 m 1-sigma (4.5 m 95%), and an assumed single-frequency ionospheric delay compensation error of 2.5 m 1-sigma (4.9 m 95%) under benign ionosphere conditions in the mid latitudes; or a total UERE (user equivalent range error) of 5.2 m 1-sigma (10.3 m 95%), are: 9.4 m 95% Horizontal Position Accuracy and 17.3 m 95% Vertical Position Accuracy. (Refer to *Global Positioning System Standard Positioning Service Performance Standard, 4th Edition, September 2008*, which may be accessed at <http://pnt.gov/public/docs/> for more detailed information.). Some of the more significant components of the error budget include:

- Receiver and antenna quality and type--signal processing characteristics
- Receiver platform dynamics--static or dynamic
- Reference frames--satellite and user
- Geographic location of user--user latitude and longitude

Satellite configuration relative to user  
Satellite characteristics--frequency stability and health  
Satellite constellation and service availability  
Satellite-User range determination accuracy  
Atmospheric conditions--signal propagation delays in ionosphere and troposphere  
Solar flux density--11-year solar cycle  
Observation length  
Multipath conditions at receiver  
Receiver noise  
Receiver mask angles  
Position computation solution algorithms

In general, there are two main components that determine the accuracy of a GPS position solution:

Geometric Dilution of Precision (GDOP)  
User Equivalent Range Error (UERE)

GDOP is the geometric effect of the spatial relationship of the satellites relative to the user. In surveying terms, it is the "strength of figure" of the trilateration position computation. GDOP varies rapidly with time since the satellites are moving. UERE is the accuracy of the individual range measurement to each satellite. UERE also varies between different satellites, atmospheric conditions, and receivers. The absolute range accuracies obtainable from absolute/autonomous GPS are largely dependent on which code (C/A or P-Code) is used to determine positions. These range accuracies (UERE), when coupled with the geometrical relationships of the satellites during the position determination (GDOP), result in a 3-D confidence ellipsoid that depicts uncertainties in all three coordinates. Given the continuously changing satellite geometry, and other factors, GPS accuracy is time/location dependent. Error propagation techniques are used to define nominal accuracy statistics for a GPS user.

4-5. Positional Accuracy Statistics--Root Mean Square. Two-dimensional (2-D) horizontal GPS positional accuracies are normally estimated and reported using a root mean square (RMS) radial error statistic. RMS error measures are approximations to error ellipses that are computed for measured points. This RMS error statistic is related to (and derives from) the positional variance-covariance matrix, which is described more fully in Chapter 11. RMS statistics can have varying confidence levels. A 1- $\sigma$  RMS error equates to the radius of a circle in which there is a 63% probability that the computed position is within this area. A circle of twice this radius (i.e. 2- $\sigma$  RMS or 2DRMS) represents (approximately) a 98 percent positional probability circle. This 98 percent probability circle, or 2DRMS, is a common positional accuracy statistic used by GPS manufacturers. In some instances, a 3DRMS, or 99+ percent probability is used. The Federal Geographic Data Committee (FGDC) and the Corps of Engineers require horizontal and vertical geospatial accuracies to be reported at the 95% RMS confidence level. For all practical purposes, the 95% RMS and 2DRMS statistics are equivalent (Note also that a RMS error statistic represents the radius of a circle and therefore is not preceded by a  $\pm$  sign.)

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a. Probable error measures. 3-D GPS accuracy measurements are sometimes expressed by Spherical Error Probable, or SEP. This measure represents the radius of a sphere with a 50% confidence or probability level. This spheroid radial measure only approximates the actual 3-D ellipsoid representing the uncertainties in the geocentric coordinate system. In 2-D horizontal positioning, a Circular Error Probable (CEP) statistic is commonly used, particular in military targeting. CEP represents the radius of a circle containing a 50% probability of position confidence.

b. Accuracy comparisons. It is important that GPS accuracy measures clearly identify the statistic from which they are derived. A "100-meter" or "3-meter" accuracy statistic is meaningless unless it is identified as being either 1-D, 2-D, or 3-D, along with the applicable probability or confidence level. For example, if a nominal SPS 2-D accuracy is specified as 7 meters CEP (i.e. 50%), then this equates to 15 meters at the 95% 2-D confidence level, and roughly 13.5 meters SEP (3-D 50%). See Table 4-1 for a comparison of the most commonly used error statistics. In addition, absolute/autonomous GPS point positioning accuracies are defined relative to an earth-centered coordinate system/datum--WGS84. This coordinate system may differ significantly from the user's local project or construction datum. Thus, any position derived from GPS observations is dependent on the accuracy of the reference datum/frame relative to WGS84. Nominal GPS accuracies may also be published as design or tolerance limits and accuracies achieved can differ significantly from these values.

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Table 4-1. Representative Statistics used in Geospatial Positioning

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Error Measurement Statistic	Probability (%)	Relative Distance ( $\sigma$ ) <sup>(1)</sup>	Nominal SPS Point Positioning Accuracy meters <sup>(2)</sup>	
<u>LINEAR MEASURES</u>			<u><math>\sigma_N</math> or <math>\sigma_E</math></u>	<u><math>\sigma_U</math></u>
Probable Error	50	0.6745 $\sigma$	± 4 m	± 9 m
Average Error	57.51	0.7979 $\sigma$	± 5 m	± 11 m
One-Sigma Standard Error/Deviation	68.27	1.00 $\sigma$	± 6.3 m	± 13.8 m
90% Probability (Map Accuracy Standard)	90	1.645 $\sigma$	± 10 m	± 23 m
95% Probability/Confidence <sup>(3)</sup>	95	1.96 $\sigma$	± 12 m	± 27 m
2-Sigma Standard Error/Deviation	95.45	2.00 $\sigma$	± 12.6 m	± 27.7 m
99% Probability/Confidence	99	2.576 $\sigma$	± 16 m	± 36 m
3-Sigma Standard Error (Near Certainty)	99.73	3.00 $\sigma$	± 19 m	± 42 m

(Table 4-1 continued on next page)

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Table 4-1. Representative Statistics used in Geospatial Positioning (continued)

<u>TWO-DIMENSIONAL MEASURES</u>			<u>CIRCULAR RADIUS</u>
1-Sigma Standard Error Circle ( $\sigma_c$ ) <sup>(4)</sup>	39	1.00 $\sigma_c$	6 m
Circular Error Probable (CEP) <sup>(5)</sup>	50	1.177 $\sigma_c$	7 m
1 Deviation Root Mean Square (1DRMS) <sup>(6)</sup>	63	1.414 $\sigma_c$	9 m
Circular Map Accuracy Standard	90	2.146 $\sigma_c$	13 m
95% 2-D Positional Confidence Circle	95	2.447 $\sigma_c$	15 m
2-Dev. Root Mean Square Error (2DRMS) <sup>(7)</sup>	98 <sup>+</sup>	2.83 $\sigma_c$	17.8 m
99% 2-D Positional Confidence Circle	99	3.035 $\sigma_c$	19 m
3.5 Sigma Circular Near-Certainty Error	99.78	3.5 $\sigma_c$	22 m
3 Dev. Root Mean Square Error (3DRMS)	99.9 <sup>+</sup>	4.24 $\sigma_c$	27 m
<u>THREE-DIMENSIONAL MEASURES</u>			<u>SPHERICAL RADIUS</u>
1- $\sigma$ Spherical Standard Error ( $\sigma_s$ ) <sup>(8)</sup>	19.9	1.00 $\sigma_s$	9 m
Spherical Error Probable (SEP) <sup>(9)</sup>	50	1.54 $\sigma_s$	13.5 m
Mean Radial Spherical Error (MRSE) <sup>(10)</sup>	61	1.73 $\sigma_s$	16 m
90% Spherical Accuracy Standard	90	2.50 $\sigma_s$	22 m
95% 3-D Confidence Spheroid	95	2.70 $\sigma_s$	24 m
99% 3-D Confidence Spheroid	99	3.37 $\sigma_s$	30 m
Spherical Near-Certainty Error	99.89	4.00 $\sigma_s$	35 m

## NOTES:

MOST COMMONLY USED STATISTICS SHOWN IN BOLD  
 ESTIMATES NOT APPLICABLE TO DIFFERENTIAL GPS POSITIONING  
 CIRCULAR/SPHERICAL ERROR RADII DO NOT HAVE  $\pm$  SIGNS

Absolute/autonomous positional accuracies are derived from GPS simulated user range errors/deviations and resultant geocentric coordinate (X-Y-Z) solution covariance matrix, as transformed to a local datum (N-E-U or  $\phi$ - $\lambda$ -h). GPS accuracy will vary with GDOP, UERE, and other numerous factors at time(s) of observation. The 3-D covariance matrix yields an error ellipsoid. Transformed ellipsoidal dimensions given (i.e.  $\sigma_N$ -  $\sigma_E$ -  $\sigma_U$ ) are only average values observed under nominal GDOP conditions. Circular (2-D) and spherical (3-D) radial measures are only approximations to this ellipsoid, as are probability estimates.

(Table 4-1 continued on next page)

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 Table 4-1. Representative Statistics used in Geospatial Positioning (continued)
 

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- (1) Valid for 2-D & 3-D only if  $\sigma_N = \sigma_E = \sigma_U$ . ( $\sigma_{\min}/\sigma_{\max}$ ) generally must be  $\geq 0.2$ . Relative distance used unless otherwise indicated.
- (2) Representative accuracy based on nominal (assumed) SPS 1-D accuracies shown in italics, and that  $\sigma_N \approx \sigma_E$ . SPS may have significant short-term variations from these nominal values. In table,  $\sigma_N = \sigma_E = 6.3$  m and  $\sigma_U = 13.8$  m.
- (3) FGDC reporting statistic for positions, elevations and depths, including USACE hydrographic survey position and depth measurement accuracy criteria.
- (4)  $\sigma_c \approx 0.5 (\sigma_N + \sigma_E)$  -- approximates standard error ellipse
- (5)  $CEP \approx 0.589 (\sigma_N + \sigma_E) \approx 1.18 \sigma_c$
- (6)  $1DRMS \approx (\sigma_N^2 + \sigma_E^2)^{1/2}$
- (7)  $2DRMS \approx 2 (\sigma_N^2 + \sigma_E^2)^{1/2}$
- (8)  $\sigma_s \approx 0.333 (\sigma_N + \sigma_E + \sigma_U)$
- (9)  $SEP \approx 0.513 (\sigma_N + \sigma_E + \sigma_U)$
- (10)  $MRSE \approx (\sigma_N^2 + \sigma_E^2 + \sigma_U^2)^{1/2}$

Source: Army Geospatial Center

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4-6. GPS Range Error Budget. There are numerous sources of measurement error that influence GPS performance. The sum of all systematic errors or biases contributing to the measurement error is referred to as range bias. The observed GPS range, without removal of biases, is referred to as a biased range--i.e. the "pseudorange." Principal contributors to the final range error that also contribute to overall GPS error are ephemeris error, satellite clock and electronics inaccuracies, tropospheric and ionospheric refraction, atmospheric absorption, receiver noise, and multipath effects. Other errors may include those that were deliberately induced by DoD before 2000--Selective Availability (S/A). In addition to this major error, GPS also contains random observation errors, such as unexplainable and unpredictable time variation. These errors are impossible to model and correct. The following paragraphs discuss errors associated with absolute/autonomous GPS positioning modes. Many of these errors are either eliminated or significantly minimized when GPS is used in a differential mode. This is due to the same errors being common to both receivers during simultaneous observing sessions. For a more detailed analysis of these errors, consult (DoD 2001) or one of the technical references listed in Appendix A.



a. Ephemeris errors and orbit perturbations. Satellite ephemeris errors are errors in the prediction of a satellite position which may then be transmitted to the user in the satellite data message. Typically these errors are less than 8 m (95%). Ephemeris errors are satellite dependent and very difficult to completely correct and compensate for because the many forces acting on the predicted orbit of a satellite are difficult to measure directly. Because direct measurement of all forces acting on a satellite orbit is difficult, it is nearly impossible to accurately account or compensate for those error sources when modeling the orbit of a satellite. Ephemeris errors produce equal error shifts in calculated absolute/autonomous point positions. More accurate satellite orbit data can be obtained at later periods for post-processing; however, this is not practical for real-time point positioning applications.

b. Clock stability. GPS relies very heavily on accurate time measurements. GPS satellites carry rubidium and cesium time standards that are usually accurate to 1 part in  $10^{12}$  and 1 part in  $10^{13}$ , respectively, while most receiver clocks are actuated by a quartz standard accurate to 1 part in  $10^8$ . A time offset is the difference between the time as recorded by the satellite clock and that recorded by the receiver. Range error observed by the user as the result of time offsets between the satellite and receiver clock is a linear relationship and can be approximated by the following formula:

$$R_E = T_O \cdot c \quad (\text{Eq 4-7})$$

where

$R_E$  = range error due to clock instability

$T_O$  = time offset

$c$  = speed of light

(1) The following example shows the calculation of the user equivalent range error (UERE)

$$T_O = 1 \text{ microsecond } (\mu\text{s}) = 10^{-6} \text{ seconds (s)}$$

$$c = 299,792,458 \text{ m/s}$$

From Equation 4-7:

$$R_E = (10^{-6} \text{ s}) * 299,792,458 \text{ m/s} = 299.79 \text{ m} \approx 300 \text{ m}$$

(2) In general, unpredictable transient situations that produce high-order departures in clock time can be ignored over short periods of time. Even though this may be the case, predictable time drift of the satellite clocks is closely monitored by the ground control stations. Through closely monitoring the time drift, the ground control stations are able to determine second-order polynomials which accurately model the time drift. The second-order polynomial determined by the ground control station to model the time drift is included in the broadcast message in an effort to keep this drift to within 1 millisecond (ms). The time synchronization between the GPS satellite clocks is kept to within 20 nanoseconds (ns) through the broadcast clock corrections as determined by the ground control stations and the synchronization of GPS standard time to the Universal Time Coordinated (UTC) to within 100 ns. Random time drifts are unpredictable, thereby making them impossible to model.

(3) GPS receiver clock errors can be modeled in a similar manner to GPS satellite clock errors. In addition to modeling the satellite clock errors and in an effort to remove them, an additional satellite should be observed during operation to simply solve for an extra clock offset parameter along with the required coordinate parameters. This procedure is based on the assumption that the clock bias is independent at each measurement epoch. Rigorous estimation of the clock terms is more important for point positioning than for differential positioning. Many of the clock terms cancel when the position equations are formed from the observations during a differential survey session.

c. Ionospheric delays. GPS signals are electromagnetic signals and as such are nonlinearly dispersed and refracted when transmitted through a highly charged environment like the ionosphere--Figure 4-3. Dispersion and refraction of the GPS signal is referred to as an ionospheric range effect because dispersion and refraction of the signal results in an error in the GPS range value. Ionospheric range effects are frequency dependent.

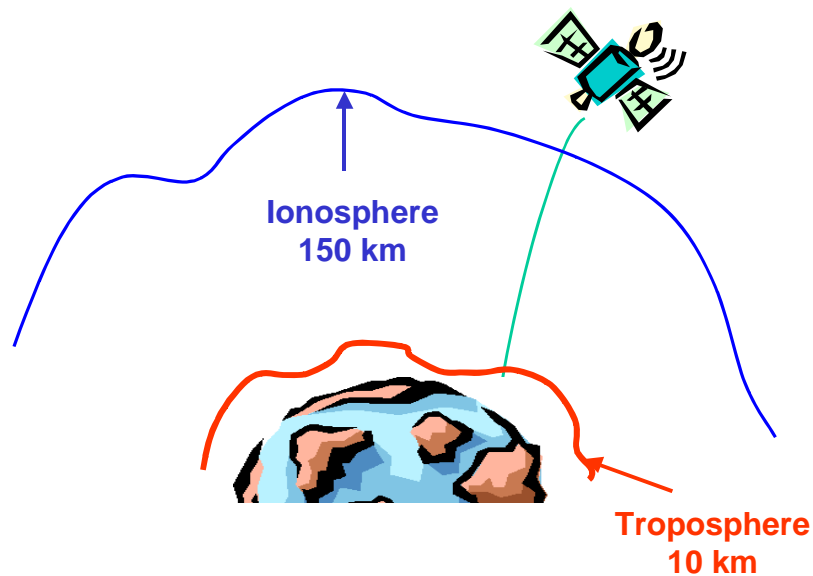


Figure 4-3. Atmospheric delays in received GPS signals

(1) The error effect of ionosphere refraction on the GPS range values is dependent on sunspot activity, time of day, and satellite geometry. Ionospheric delay can vary from 40-60 m during the day and 6-12 m at night. GPS operations conducted during periods of high sunspot activity or with satellites near the horizon produce range results with the most error. GPS operations conducted during periods of low sunspot activity, during the night, or with a satellite near the zenith produce range results with the least amount of ionospheric error.

(2) Resolution of ionospheric refraction can be accomplished by use of a dual-frequency receiver (a receiver that can simultaneously record both L1 and L2 frequency measurements).

During a period of uninterrupted observation of the L1 and L2 signals, these signals can be continuously counted and differenced, and the ionospheric delay uncertainty can be reduced to less than 5 m. The resultant difference reflects the variable effects of the ionosphere delay on the GPS signal. Single-frequency receivers used in an absolute/autonomous and differential positioning mode typically rely on ionospheric models that model the effects of the ionosphere. Significant ionospheric delay removal can be achieved using dual-frequency receivers.

(3) L-band signals transmitted by the GPS satellites are delayed as they travel through the ionosphere and the troposphere. The residual effects remaining after between-station differencing are generally small for short baselines, and hence are often neglected. For longer interstation distances the ionospheric and tropospheric effects cannot be ignored because the unmodelled biases begin to degrade the solution by an appreciable amount (the magnitude of its effect tends to grow in proportion to the baseline length), and they make the ambiguity resolution process more difficult because they tend to "destroy" the integer nature of the ambiguity parameters. The ionospheric delay is highly unpredictable, being a function of the latitude of the receiver, the elevation angle to the satellite, the time-of-day of the observations, and the level of solar activity at the time of observation (itself dependent on a number of factors such as the season, the 11 year solar cycle, etc.). Note that the time delay is proportional to the inverse of the frequency squared. That is, higher frequencies are less affected by the ionosphere, and hence the ionospheric time delay for L1 observations (1575.42MHz) is less than for L2 observations (1227.60MHz), which is less than for L5 observations (1176.45MHz). Hence the L2 ionospheric effect is approximately 1.646 times that on L1 ( $1.646 \approx f_1^2 / f_2^2$ ) and the L5 ionospheric effect is approximately 1.793 times that on L1 ( $1.646 \approx f_1^2 / f_5^2$ )  
(substantially excerpted from  
[http://www.gmat.unsw.edu.au/snap/gps/gps\\_survey/principles\\_gps.htm](http://www.gmat.unsw.edu.au/snap/gps/gps_survey/principles_gps.htm))

d. Tropospheric delays. GPS signals in the L-band level are not dispersed by the troposphere, but they are refracted due to moisture in the lower atmosphere. The tropospheric conditions causing refraction of the GPS signal can be modeled by measuring the dry and wet components. The dry component is best approximated by the following equation:

$$D_C = (2.27 \cdot 0.001) \cdot P_O \quad (\text{Eq 4-8})$$

where

$D_C$  = dry term range contribution in zenith direction in meters

$P_O$  = surface pressure in millibar (mb)

(1) The following example shows the calculation of average atmospheric pressure  $P_O$  = 1013.243 mb:

From Equation 4-8:

$$\begin{aligned} D_C &= (2.27 \cdot 0.001) \cdot 1013.243 \text{ mb} \\ &= 2.3 \text{ m, the dry term range error contribution in the zenith direction} \end{aligned}$$

(2) The wet component is considerably more difficult to approximate because its approximation is dependent not just on surface conditions, but also on the atmospheric conditions (water vapor content, temperature, altitude, and angle of the signal path above the horizon) along the entire GPS signal path. As this is the case, there has not been a well-correlated model that approximates the wet component.

(3) Tropospheric delay is caused by the signal refraction in the electrically *neutral* (or non-ionised) atmospheric layer called the troposphere, extending from the earth's surface to about 8km (though approximately twice as thick at the equator). Another component of the neutral atmosphere is the stratosphere, extending up to an altitude of about 50km, to the base of the ionospheric layer. For the purposes of discussing neutral atmospheric delay, under the term "tropospheric delay" will be included both the components due to the troposphere and the stratosphere because the troposphere, although being relatively thin, contains most of the mass of the neutral atmosphere and practically all of the water vapor. Tropospheric delay is a function of the satellite elevation angle and the altitude of the receiver, and is dependent on the atmospheric pressure, temperature, and water vapor pressure. About 90% of the magnitude of the tropospheric delay arises from the dry component, and the remaining 10% from the wet component. With respect to microwaves up to frequencies of 15GHz the neutral atmosphere is a non-dispersive medium, and hence the tropospheric delay is not frequency-dependent. It cannot therefore be eliminated through linear combinations of L1, L2 and/or L5 observations as in the case of the ionospheric delay. The magnitude of the tropospheric delay is the same for L1, L2 and L5 observations, and for pseudo-range and carrier phase measurements. There is significantly less tropospheric delay at high altitude than at sea level. For example, if two receivers are deployed, one at sealevel and the other at 3000m, and assuming a satellite elevation angle of 20°, the tropospheric refraction error for the receiver at sealevel will be approximately 7.8m, while for the receiver positioned at high altitude the tropospheric refraction error is of the order of 4.8m.

*(substantially excerpted from*

[http://www.gmat.unsw.edu.au/snap/gps/gps\\_survey/principles\\_gps.htm](http://www.gmat.unsw.edu.au/snap/gps/gps_survey/principles_gps.htm))

e. **Multipath.** Multipath describes an error affecting positioning that occurs when the signal arrives at the receiver from more than one path--see Figure 4-4. Multipath normally occurs near large reflective surfaces, such as a metal building or structure. GPS signals received as a result of multipath give inaccurate GPS positions when processed. With the newer receiver and antenna designs, and sound prior mission planning to reduce the possible causes of multipath, the effects of multipath as an error source can be minimized. Averaging of GPS signals over a period of time (i.e. different satellite configurations) can also help to reduce the effects of multipath.

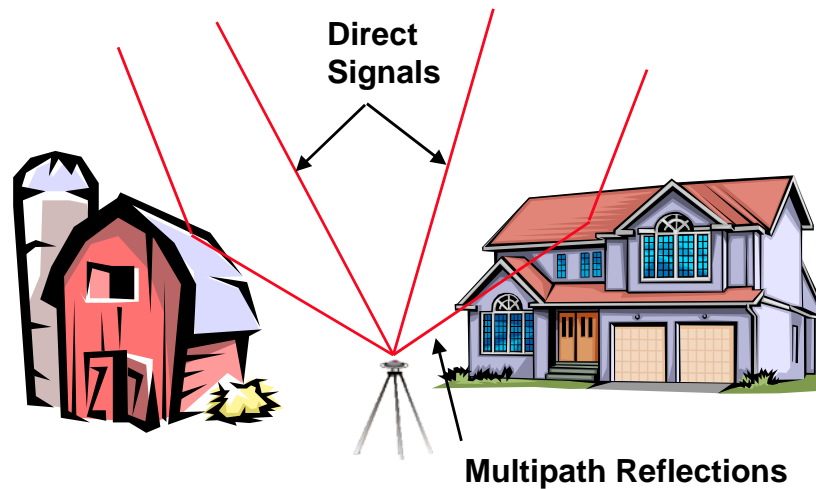


Figure 4-4. Multipath signals impacting GPS observations

f. Receiver noise. Receiver noise includes a variety of errors associated with the ability of the GPS receiver to measure a finite time difference. These include signal processing, clock/signal synchronization and correlation methods, receiver resolution, signal noise, and others.

g. Selective Availability (S/A) and Anti-Spoofing (A/S). Before 2000, S/A was activated to purposely degrade the satellite signal to create position errors. This is done by dithering the satellite clock and offsetting the satellite orbits. Prior to 2000, the effects of S/A were eliminated by using differential techniques. DoD always reserves the right to reimplement S/A should a major military conflict require this action for national security. However, it is the stated intent of the US Government not to implement S/A globally but to develop regional GPS denial capabilities that will not impact GPS users globally. A/S is implemented by interchanging the P code with a classified Y code. This denies users who do not possess an authorized decryption device. Manufacturers of civil GPS equipment have developed methods such as squaring or cross correlation in order to make use of the P code when it is encrypted.

4-7. User Equivalent Range Error. The previous sources of errors or biases are principal contributors to overall GPS range error. There are many others in the total error budget model. This total error budget is often summarized as the User Equivalent Range Error (UERE), or as User Range Error (URE). To distinguish between the satellite-dependent errors and that of the user's receiver, a Signal-in-Space (SIS) URE is defined by the DoD. This SIS URE does not include the receiver's noise and multipath effects. As mentioned previously, many of these range errors can be removed or at least effectively suppressed by developing models of their functional relationships in terms of various parameters that can be used as a corrective supplement for the basic GPS information. Differential techniques also eliminate many of these errors. Table 4-2 lists the more significant error sources for a single-frequency receiver, as observed globally by DoD on the given date. The resultant URE does not include multipath effects.

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Table 4-2. Estimate of Standard Positioning System User Range Error  
Single Frequency Receiver (8 June 2000)

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Error Source	User Range Error Contribution (± meters)
Navigation Message Curve Fit	0.20
Tropospheric Model	0.25
C/A Code Phase Bias	0.27
Orbit	0.57
Receiver Noise	0.80
Satellite Clock	1.43
Ionospheric Model (global average)	7.00 <sup>1</sup>
<hr/>	
URE (95%)	± 7.22 m

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<sup>1</sup> Ionospheric model ranged from 1.30 m (best) to 7.00 m (worst)

Source: Figure A-5-12, (DoD 2001)

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Globally, the URE for a single frequency ranged from 2.2 m to 14.6 m. A dual-frequency receiver had a far more accurate URE: 1.4 m to 2.3 m, with a global average of 1.7 m. If receiver multipath and other effects are added, say ± 2 to 4 m, then the UERE for a single-frequency receiver would be in the 10-15 m range.

4-8. Satellite Geometry Effects on Accuracy-- Dilution of Precision. The final positional accuracy of a point determined using absolute/autonomous GPS survey techniques is directly related to the geometric strength of the configuration of satellites observed during the survey session. GPS errors resulting from satellite configuration geometry can be expressed in terms of GDOP. In mathematical terms, Geometric DOP is a scalar, dimensionless quantity used in an expression of a ratio of the positioning accuracy. It is the ratio of the standard deviation of one coordinate to the measurement accuracy. GDOP represents the geometrical contribution of a certain scalar factor to the uncertainty (i.e. standard deviation) of a GPS measurement. GDOP values are a function of the diagonal elements of the covariance matrices of the adjusted parameters of the observed GPS signal and are used in the point formulations and determinations.

a. In a more practical sense, GDOP is a scalar quantity of the contribution of the configuration of satellite constellation geometry to the GPS accuracy, in other words, a measure of the "strength" of the geometry of the satellite configuration. In general, the more satellites that can be observed and used in the final solution, the better the solution. Since GDOP can be used as a measure of the geometrical strength, it can also be used to selectively choose four satellites in a particular constellation that will provide the best solution. Satellites spread around the horizon will provide the best horizontal position, but the weakest vertical elevation.

Conversely, if all satellites are at high altitudes, then the precision of the horizontal solution drops but the vertical improves. This is illustrated in Figure 4-5. The smaller the GDOP, the more accurate the position.

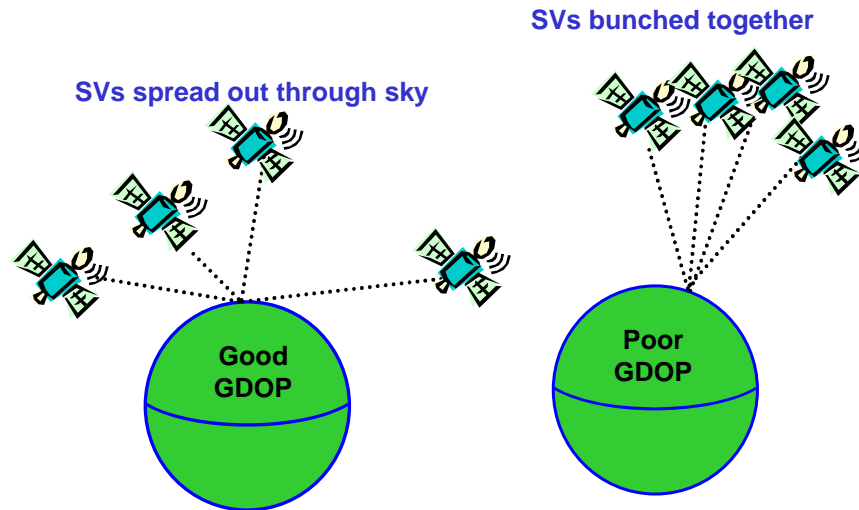


Figure 4-5. Satellite geometry and GDOP--"Good" GDOP and "Poor" GDOP configurations  
Using receivers that track GLONASS as well as GPS minimizes this problem

b. GDOP values used in absolute/autonomous GPS positioning is a measure of spatial accuracy of a 3-D position and time. The GDOP is constantly changing as the relative orientation and visibility of the satellites change. GDOP can be computed in the GPS receivers in real-time, and can be used as a quality control indicator. GDOP is defined to be the square root of the sum of the variances of the position and time error estimates.

$$\text{GDOP} = [\sigma_E^2 + \sigma_N^2 + \sigma_U^2 + \sigma_R^2 + (c \cdot \delta_T)^2]^{0.5} \cdot [1 / \sigma_R] \quad (\text{Eq 4-9})$$

where

$\sigma_E$  = standard deviation in east value, m

$\sigma_N$  = standard deviation in north value, m

$\sigma_U$  = standard deviation in up direction, m

$c$  = speed of light (299,338,582.7 m/s)

$\delta_T$  = standard deviation in time, seconds

$\sigma_R$  = overall standard deviation in range in meters, i.e. the UERE at the one-sigma (68%) level

The GDOP value is easily estimated by assuming the UEREs are all unity and then pulling the standard deviations directly from the variance-covariance matrix of the position adjustment. Thus GDOP (and its derivations) can be recomputed at each position update (e.g., every second). Large jumps (increases) in GDOP values are poor performance indicators, and typically occur as satellites are moved in and out of the solution.

c. Positional dilution of precision (PDOP). PDOP is a measure of the accuracy in 3-D position, mathematically defined as:

$$\text{PDOP} = [\sigma_E^2 + \sigma_N^2 + \sigma_U^2]^{0.5} \cdot [1 / \sigma_R] \quad (\text{Eq 4-10})$$

where all variables are equivalent to those used in Equation 4-9. PDOP is simply GDOP less the time bias.

(1) PDOP values are generally developed from satellite ephemerides prior to conducting a survey. When developed prior to a survey, PDOP can be used to determine the adequacy of a particular survey schedule.

(2) The key to understanding PDOP is to remember that it represents position recovery at an instant in time and is not representative of a whole session of time. When using pseudorange techniques, PDOP values in the range of 4-5 are considered very good, while PDOP values greater than 10 are considered very poor. For static surveys it is generally desirable to obtain GPS observations during a time of rapidly changing GDOP and/or PDOP.

(3) When the values of PDOP or GDOP are viewed over time, peak or high values (>10) can be associated with satellites in a constellation of poor geometry. The higher the PDOP or GDOP, the poorer the solution for that instant in time. This is critical in determining the acceptability of real-time navigation and photogrammetric solutions. Poor geometry can be the result of satellites being in the same plane, orbiting near each other, or at similar elevations.

d. Horizontal dilution of precision (HDOP). HDOP is a measurement of the accuracy in 2-D horizontal position, mathematically defined as:

$$\text{HDOP} = [\sigma_E^2 + \sigma_N^2]^{0.5} \cdot [1 / \sigma_R] \quad (\text{Eq 4-11})$$

This HDOP statistic is most important in evaluating GPS surveys intended for densifying horizontal control in a project. The HDOP is basically the RMS error determined from the final variance-covariance matrix divided by the standard error of the range measurements. HDOP roughly indicates the effects of satellite range geometry on a resultant position.

e. Vertical dilution of precision (VDOP). VDOP is a measurement of the accuracy in standard deviation in vertical height, mathematically defined as:

$$\text{VDOP} = [\sigma_U] \cdot [1 / \sigma_R] \quad (\text{Eq 4-12})$$

f. Acceptable DOP values. In general, GDOP and PDOP values should be less than 6 for a reliable solution. Optimally, they should be less than 5. GPS performance for HDOP is normally in the 2 to 3 range. VDOP is typically around 3 to 4. Increases above these levels may indicate less accurate positioning. In most cases, VDOP values will closely resemble PDOP values. It is also desirable to have a GDOP/PDOP that changes during the time of GPS survey session. The lower the GDOP/PDOP, the better the instantaneous point position solution is.



4-9. Resultant Positional Accuracy of Point Positioning. The relationship between positional solution, the range error, and DOP can be expressed as follows (Leick, 1995):

$$\text{Positional solution } (\sigma) = \sigma_R \cdot \text{DOP} \quad (\text{Eq 4-13})$$

where

$\sigma$  = horizontal or vertical positional accuracy

$\sigma_R$  = range error (95% UERE)

For example, if the observed HDOP of a point position is displayed as 2.0 assuming unity *a priori* deviations, and the estimated 95% UERE is 4 m, then the estimated horizontal positional accuracy would be 8 m. Since the UERE and HDOP (PDOP/HDOP/VDOP) values are so variable over short periods of time, there is little practical use in estimating a positional accuracy in this manner. Positional accuracy is best estimated by statistically comparing continuous observations at some known reference point, typically over a 24-hour period, and computing the 95% deviations.

a. From actual DoD worldwide observations, the results of actual horizontal and vertical positional accuracies of single- and dual-frequency GPS point positioning observed on two different dates are summarized in Table 4-3.

Table 4-3. GPS All-in-View Performance--95%

	Single Frequency		Dual Frequency	
	Horizontal m	Vertical m	Horizontal m	Vertical m
3 June 2000 <sup>1</sup>				
Global average	8.3	16.8	3.1	5.6
Worst site	19.7	44.0	5.0	9.2
8 June 2000 <sup>1</sup>				
Global average	7.8	16.2	2.6	4.3
Worst site	19.2	39.3	4.2	7.1
Predictable Accuracy <sup>2</sup> 13		22	--	--
Worst case	36	77		

Sources:

1 Tables A-5-1 through A-5-4 (DoD 2001)

2 2001 Federal Radionavigation Plan/Systems Predicted Accuracy (FRS Table 3-1--GPS System Characteristics)

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b. Table 4-3 shows that single-frequency receivers are capable of achieving around 10 m (95%) positional accuracy and that the vertical component is significantly poorer. The 2001 Federal Radionavigation Plan/System (FRP 2001) advertises a predictable SPS accuracy of 13 m (horizontal) and 22 m (vertical), with a global service availability of 99%. This predictable accuracy estimate does not include error contributions due to ionospheric contributions, tropospheric contributions, or receiver noise. There would be few applications for using GPS point positioning methods for elevation determination given the 20+ m error. The results also clearly show the accuracy improvements when dual-frequency receivers are used. There are many GIS database development applications where a horizontal accuracy in the 10 to 30 m range is sufficiently accurate; thus point positioning with a single- or dual-frequency receiver is a reliable, fast, and economical procedure for those applications. These point positioning accuracy levels are obviously not suitable for USACE design and construction purposes; thus, relative or differential positioning techniques are required.

c. For a more complete, comprehensive, and up-to-date discussion and explanation of Point Positioning accuracy issues, please refer to *Global Positioning System Standard Positioning Service Performance Standard, 4th Edition, September 2008*, which may be accessed at <http://pnt.gov/public/docs/>.